QP CODE : 24396

(3 Hours) **Marks: 80**

Note: 1. Question no. **1** is compulsory.
2. Attempt any **three** questions out of remaining **five** questions.

Q.1.[a] Determine the constants a, b, c, d so that the function [5]

$$f(z)=x^2+axy+by^2+i(cx^2+dxy+y^2) \text{ is analytic.}$$

[b] Let $A=\{1,2,3,4\}, B=\{1,2,3,4\}$ and "aRb if and only if a is not equal to b". Find R and its digraph. [5]

[c] For the sets A, B, C given that $A\cap B=A\cap C$ and $\bar{A}\cap B=\bar{A}\cap C$. Is it necessary that B = C? Justify. [5]

[d] Find Laplace transform of [5]

$$f(t)= t \quad \text{for } 0<t<1$$
$$= 0 \quad \text{for } 1<t<2, \quad f(t+2)=f(t).$$

Q.2.[a] 75 Children went to an amusement park where they can ride on the merry-go-round, roller coaster and ferris wheel. It is known that 20 of them have taken all 3 rides, and 55 of them have taken at least two of the 3 rides. Each ride costs 0.50 Rs and the total receipt of the amusement park was 70 Rs. Determine the number of children who did not try any of the rides. [6]

[b] Evaluate [6]

$$\int_0^\infty te^{-3t}J_0(4t)dt=\frac{3}{125} \text{ if } L\{J_0(t)\}=\frac{1}{\sqrt{s^2+1}}.$$

[c] (i) Functions f , g and h are defined as follows : [4]

$f:R\rightarrow R$, $g:R\rightarrow R$, $h:R\rightarrow R$, $f(x)=x+4$, $g(x)=x-4$

$h(x)=4x$ for $x\in R$, where R is the set of real numbers.

Compute $f\circ g$; $g\circ f$; $f\circ g\circ h$; $h\circ h$.

(ii) Show that using Venn diagram $P\cap(Q-R)=(P\cap Q)-(P\cap R)$. [4]

Q.3.[a] If $f(z)$ and $|f(z)|$ are both analytic then show that $f(z)$ is constant. [6]

[b] Let R be a binary relation on the set of positive integers such that [6]

R = {(a,b) / a-b is an odd positive integer }. Is R reflexive ?

Symmetric ? Antisymmetric ? Transitive ? An equivalence relation ?

A partial ordering set ?

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[c] Evaluate (i) $L[te^{3t}\sin 4t]$ (ii) $L\left[\int_0^t\int_0^t\int_0^t t\sin t\,dt\,dt\,dt\right]$ [8]

Q.4. [a] Evaluate using Convolution theorem $L^{-1}\left[\frac{(s+2)}{(s^2+4s+8)^2}\right]$. [6]

[b] Find the transitive closure of R where R be the relation [6]

represented by $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

[c] Find analytic function $f(z) = u + iv$ where $v = e^x(x\sin y + y\cos y)$.. [8]

Q.5.[a] Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ with $y(0) = 1$. [6]

[b] Find bilinear transformation which maps the points z = 1,i,-1 onto $w = 0,1,\infty$. Further show that under this transformation the unit circle In w plane is mapped onto a straight line in the z plane.

[c] In a bolt factory machines A, B, and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4, 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C? [8]

Q.6. [a] It is known that at the university 60% of the professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog, 40% play bridge and jog. If someone claimed that 20% of the professors jog and play bridge and tennis, would you believe this claim? Why? [6]

[b] Suppose repetitions are not permitted. [6]

(i) How many four- digit numbers can be formed from the digits 1, 2, 3, 5, 7, 8?

(ii) How many of the numbers in part (a) are less than 4000?

(iii) How many of the numbers in part (a) are odd ?

(iv) How many of the numbers in part (a) are multiples of 5?

[c] Evaluate (i) $L^{-1}\left[2\tanh^{-1} s\right]$ (ii) $L^{-1}\left[\frac{e^{4-3s}}{(s+4)^{\frac{5}{2}}}\right]$ [8]

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