

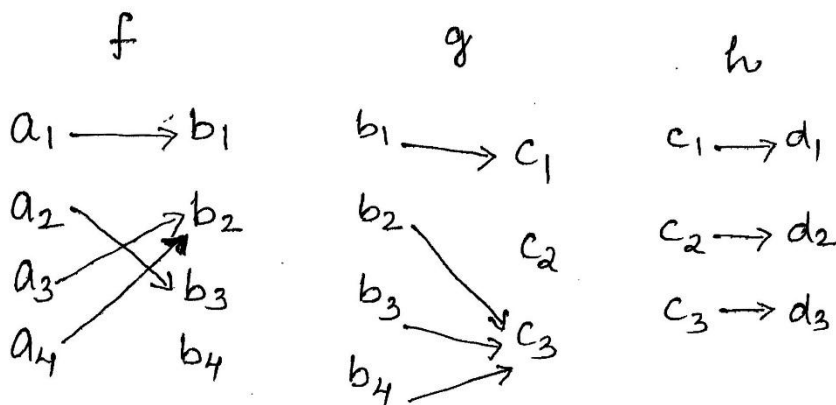
(3 Hours)

(Total Marks : 80)

- Note:** 1. Question no. 1 is compulsory.  
2. Attempt any **three** questions out of remaining **five** questions.

- Q.1.[a]** Evaluate  $L[\sin 2t \cos t \cosh 2t]$ . [5]  
**[b]** How many friends must you have to guarantee that atleast five of them have birthday in the same month. [5]  
**[c]** Determine the constants a, b, c, d, e so that the function  $f(z) = ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2 + i(4x^3y - exy^3 + 4xy)$  is analytic. [5]  
**[d]** Out of one lakh people 51500 are female and 48500 are male. Among the females 9000 are singers, among the males 30200 are singers. A person chosen randomly. If A, B, C are the events that a singer is chosen, a female is chosen and male is chosen respectively then find (i)  $P(A/B)$  (ii)  $P(A/C)$  (iii)  $P(A/C)$  (iv)  $P(C/A)$ . [5]

- Q.2. [a]** Using Venn diagram show that  $P \cap (Q \oplus R) = (P \cap Q) \oplus (P \cap R)$ . [6]  
**[b]** Evaluate  $L\{f(t)\}$  where  $f(t) = \begin{cases} 1 & 0 \leq t < a \\ -1 & a < t < 2a \end{cases}$  and  $f(t+2a) = f(t)$ . [6]  
**[c]** Let f, g, h be the functions shown in the diagram : [8]



- Find :** (i)  $g \circ f, h \circ (g \circ f), (h \circ g) \circ f, h^{-1}$   
(ii) Identify onto and one-one function for 3 of them.

- Q.3. [a]** Find analytic function  $f(z) = u + iv$  where  $v = \frac{x}{x^2 + y^2} + \cosh x \cos y$ . [6]  
**[b]** Solve  $(D^2 + 2D + 5)y = e^{-t} \sin t$ , when  $y(0) = 0, y'(0) = 1$ . [6]  
**[c]** Evaluate (i)  $L\left\{\frac{1}{t}(1 - \cos t)\right\}$  [8]  
(ii)  $\int_0^{\infty} e^{-t} \left( \int_0^t u^4 \sinh u \cosh u du \right) dt$

- Q.4. [a]** Evaluate using convolution theorem  $L^{-1}\left[\frac{(s+2)}{(s^2+4s+8)^2}\right]$  [6]
- [b]** Find bilinear transformation which maps the points  $z = -1, 1, \infty$  onto  $w = -i, -1, i$ . [6]
- [c]** Three machines A, B and C produce respectively 25%, 35% and 40% of the total number of items of a factory. The percentages of defective output of these machines are respectively 5%, 4% and 2%. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine A. [8]
- Q.5. [a]** Suppose repetitions are not permitted. [6]
- (i) How many four- digit numbers can be formed from the digits 1, 2, 3, 5, 7, 8?
- (ii) How many of the numbers in part (a) are less than 4000?
- (iii) How many of the numbers in part (a) are multiples of 5?
- [b]** Let  $A = \{1, 2, 3, 4, 12\}$  and let R be the relation on A defined by  $xRy$  if and only if "x divides y", Show that (A,R) is a PO set. Draw the diagraph of R. [6]
- [c]** Evaluate (i)  $L^{-1}\left[\frac{e^{-5s}}{(s-2)^4}\right]$  (ii)  $L^{-1}\left[\log\left(\frac{s+3}{s+5}\right)\right]$  [8]
- Q.6. [a]** It is known that at the university 60% of the professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog, 40% play bridge and jog. If someone claimed that 20% of the professors jog and play bridge and tennis, would you believe this claim? Why? [6]
- [b]** Solve  $a_{r+2} + 2 a_{r-1} - 3a_r = 0$  that satisfies  $a_0 = 1, a_1 = 2$ . [6]
- [c]** (i) If  $f(z)$  is an analytic and  $|f(z)|$  is constant, show that  $f(z)$  is constant. [8]
- (ii) Find the image of  $|z-ai| = a$  under the transformation  $w = \frac{1}{z}$ .