

(3 Hours)

[Total Marks: 80]

N.B. : 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Evaluate $\int_0^{\infty} e^{-2t} t \sin t \, dt$. (5)

(b) Find a, b, c, d, e if $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2) + i(dx^2y - 2y^3 + exy)$ is analytic. (5)

(c) Find half range sine series of $f(x) = x(\pi - x)$, $0 < x < \pi$. (5)

(d) Find directional derivative of $\phi = 4xz^2 + x^2yz$, at $(1, -2, -1)$ in direction of $2i - j - 2k$. (5)

Q.2 (a) Prove that $\nabla r^n = nr^{(n-2)} \bar{r}$. (6)

(b) Find Bilinear Transformation which maps the points $z = -1, 0, 1$ onto the points $w = -1, -i, 1$. (6)

(c) Find i) $L^{-1} \left[\frac{e^{-2s}}{s^2 + 3s + 2} \right]$ ii) $L^{-1} \left[\log \left(\frac{s^2 + 4}{s + 4} \right) \right]$. (8)

Q.3 (a) Use Gauss's Divergence Theorem to evaluate $\iiint_s \bar{N} \cdot \bar{F} \, ds$ where (6)

$\bar{F} = 4xi - 2y^2j + z^2k$ and s is region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 4$.

(b) Find Laplace Transform of $e^{-2t} \int_0^t u e^{3u} \cos 4u \, du$. (6)

(c) Obtain Fourier series of $f(x) = \begin{cases} x + \frac{\pi}{2} & -\pi < x < 0 \\ \frac{\pi}{2} - x & 0 < x < \pi \end{cases}$. (8)

Hence deduce $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$

- Q.4 (a) Show that set of functions $\left\{ \frac{\sin x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \frac{\sin 3x}{\sqrt{\pi}} \dots \right\}$ form an orthonormal set in $(-\pi, \pi)$. (6)
- (b) Find orthogonal trajectories of the family of curves $e^x \cos y - xy = c$. (6)
- (c) Prove that $\vec{F} = (6xy^2 - 2z^3)\mathbf{i} + (6x^2y + 2yz)\mathbf{j} + (y^2 - 6z^2x)\mathbf{k}$ is irrotational. Find scalar potential of \vec{F} . Hence find the work done of moving particle from (1,0,2) to (0,1,1). (8)
- Q.5 (a) Find Fourier Integral representation for $f(x) = \begin{cases} 1-x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$. (6)
- (b) Solve using Laplace Transform $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t}$ given $y(0) = 4$ and $y'(0) = 2$. (6)
- (c) Verify Green's Theorem for $\vec{F} = x^2\mathbf{i} - xy\mathbf{j}$ and c is triangle having vertices $A(0,2)$, $B(2,0)$, $C(4,2)$. (8)
- Q.6 (a) Using Convolution theorem, find Inverse Laplace of $\frac{s}{(s^2 + 4)^2}$. (6)
- (b) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{3}{x} \sin x + \frac{(3-x^2)}{x^2} \cos x \right]$. (6)
- (c) Find Fourier series for $f(x) = (\pi - x)^2$ in $0 \leq x \leq 2\pi$. Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ (8)