

Time: 3 hours

Marks: 80

N.B 1) Question No. 1 is **Compulsory**.2) **Answer** any **three** questions from remaining questions.

3) Figures to the right indicate full marks.

- Q.1 a) Evaluate  $\int_0^{\infty} \frac{e^{-x^3}}{\sqrt{x}} dx$ . 3
- b) Find the length of the curve  $x = \frac{y^3}{3} + \frac{1}{4y}$  from  $y = 1$  to  $y = 2$ . 3
- c) Solve  $(D^2 + D)y = e^{4x}$ . 3
- d) Evaluate  $\int_0^1 \int_{x^2}^x xy(x+y)dydx$ . 3
- e) *Solve* 4  
 $(4x + 3y - 4)dx + (3x - 7y - 3)dy = 0$ .
- f) Solve  $\frac{dy}{dx} = 1 + xy$  with initial condition 4  
 $x_0 = 0, y_0 = 0.2$  by Taylors series method. Find the approximate value of y for  $x=0.4$  (step size 0.4).
- Q.2 a) Solve  $\frac{d^2y}{dx^2} - 16y = x^2 e^{3x} + e^{2x} - \cos 3x + 2^x$ . 6
- b) Show that  $\int_0^{\pi} \frac{\log(1+\cos x)}{\cos x} dx = \pi \sin^{-1} a$   $0 \leq a \leq 1$ . 6
- c) Change the order of integration and evaluate  $\int_0^2 \int_{2-\sqrt{4-y^2}}^{2+\sqrt{4-y^2}} dx dy$ . 8
- Q.3 a) Evaluate  $\iiint (x + y + z) dx dy dz$  over the tetrahedron bounded by 6  
the planes  $x = 0, y = 0, z = 0$  and  $x + y + z = 1$ .

b) Find the mass of the lamina bounded by the curves  $y = x^2 - 3x$  and  $y = 2x$  if the density of the lamina at any point is given by  $\frac{24}{25}xy$ . 6

c) Solve  $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + 3y = \frac{\log x \cdot \cos(\log x)}{x}$ . 8

Q.4 a) Find by double integration the area bounded by the parabola  $y^2 = 4x$  and the line  $y = 2x - 4$ . 6

b) Solve  $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ . 6

c) Solve  $\frac{dy}{dx} = x^3 + y$  with initial conditions  $y(0) = 2$  at  $x=0.2$  in steps of  $h=0.1$  by Runge Kutta method of fourth order. 8

Q.5 a) Evaluate  $\int_0^1 x^5 \sin^{-1} x \, dx$  and find the value of  $\beta \left( \frac{9}{2}, \frac{1}{2} \right)$ . 6

b) In a circuit containing inductance L, resistance R, and voltage E, the current i is given by  $L \frac{di}{dt} + Ri = E$ . Find the current i at time t if at  $t=0, i=0$  and L, R, E are constants. 6

c) Evaluate  $\int_0^6 \frac{dx}{1+3x}$  by using i) Trapezoidal ii) Simpsons (1/3)rd and iii) Simpsons (3/8)th rule. 8

Q.6 a) Find the volume bounded by the paraboloid  $x^2 + y^2 = az$  and the cylinder  $x^2 + y^2 = a^2$ . 6

b) Change to polar coordinates and evaluate  $\int_0^1 \int_0^x (x+y) dy dx$ . 6

c) Solve by method of variation of parameters  $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{e^x}$ . 8

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