

Time:3 hours

Total marks: 80

- N.B. (1) Question No.1 is compulsory.
 (2) Answer any three questions from remaining.
 (3) Figures to the right indicate full marks.

- Q1. a) Evaluate $\int_0^{\infty} \frac{\sin 3t + \sin 2t}{te^t} dt$ 05
 b) Find the directional derivative of the function $\phi = 4xz^2 + x^2yz$ at $(1,-2,-1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. 05
 c) Expand $f(x) = \pi x - x^2$ in a half range sine series in the interval $(0,\pi)$ 05
 d) Show that the function $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Find the corresponding analytic function $f(z)$. 05

- Q2. a) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{\sin x}{x} - \cos x \right]$ 06
 b) Find Fourier series to represent $f(x) = 4 - x^2$ in the interval $(0,2)$. 06
 c) Solve the following differential equation using Laplace transform $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = 3te^{-t}$, given $y(0)=4, y'(0)=2$. 08

- Q3. a) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is conservative. Find the scalar potential for \vec{F} and also find the work done by \vec{F} in moving a particle from $(1,0,1)$ to $(2,1,3)$ 06
 b) Obtain the complex form of the Fourier series for $f(x) = e^{3x}$ in $(0,3)$ 06
 c) Find the Inverse Laplace Transform of 08
 i) $\frac{8s + 20}{s^2 - 12s + 32}$ ii) $\tan^{-1} \left(\frac{s+a}{b} \right)$

Q.4 a) Prove that $\int J_3(x)dx + 2 \frac{J_1(x)}{x} + J_2(x) = 0$ 06

b) Evaluate $\int_C (x^2 y dx + x^2 dy)$ where C is the boundary described in the anti clockwise direction of the triangle with vertices (0,0),(1,0) and (1,1). 06

c) Find Fourier series expansion of 08

$$f(x) = \begin{matrix} 2 & -2 < x < 0 \\ x & 0 < x < 2 \end{matrix}$$

Q5. a) Show that the map of the real axis of the z plane is a circle 06

under the transformation $w = \frac{2}{z+i}$. Find the centre and radius of the circle.

b) Find the Fourier Integral representation of 06

$$f(x) = \begin{matrix} 1 & |x| < 1 \\ 0 & |x| > 1 \end{matrix} \text{ hence evaluate } \int_0^{\infty} \frac{\sin \omega \cos \omega x}{\omega} d\omega$$

c) i) Find the Laplace Transform of 04

$$(1 + 2t - t^2 + t^3) H(t - 4)$$

ii) If $\vec{F} = x^2 z \hat{i} - 2y^3 z^3 \hat{j} + xy^2 z^2 \hat{k}$, find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ 04

Q6. a) Use Convolution theorem to find $L^{-1} \left\{ \frac{s^2}{(s^2 + 4)^2} \right\}$ 06

b) Use Gauss Divergence Theorem to evaluate $\iint_S \vec{N} \cdot \vec{F} ds$ where 06

$\vec{F} = 4x\hat{i} + 3y\hat{j} - 2z\hat{k}$ and S is the surface bounded by $x=0, y=0, z=0$ and $2x+2y+z=4$.

c) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and 08

$u + v = \cos x \cosh y - \sin x \sinh y$ find $f(z)$ in terms of z