 QP CODE : 40416

(3 Hours) **[Total Marks: 80]**

N.B (1) Question **No. 1** is **compulsory**.
(2) Solve any **three** questions out of remaining **five** questions.
(3) Assumptions made should be clearly stated.
(4) Figures to the right indicate full marks.

Q.1 (a) Two dice are rolled, find the probability that the sum is [6M]
(i)Equal to 1 (ii) Equal to 4 (iii) Less than 13

(b) Use the laws of logic to show that
$[(p\rightarrow q) \wedge \sim q] \rightarrow \sim p$ is a tautology [6M]

(c) Determine the matrix of the partial order of divisibility on the set A.Draw the Hasse diagram of the Poset.Indicate those which are chains [8M]
(1) A={1,2,3,5,6,10,15,30}
(2) A={3,6,12,36,72}

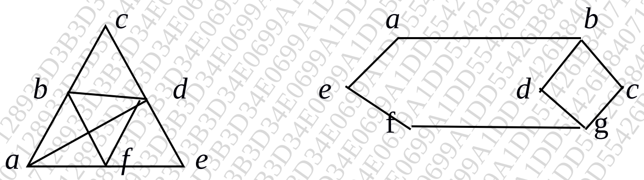
Q.2 (a) Find the complement of each element in D_{42} . [6M]

(b) Let Q be the set of positive rational numbers which can be expressed in the form $2^a 3^b$, where a and b are integers .Prove that algebraic structure (Q, **.**) is a group. Where **.** is multiplication operation. [6M]

(c) Define isomorphic graphs .Show whether the following graphs are isomorphic or not . [8M]

Fig (a) *Fig (b)*

Q.3 (a) Determine which of the following graph contains an Eulerian or Hamiltonian circuit. [6M]



Fig(a) *Fig(b)*

(b) For all sets A, X and Y show that
$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$ [6M]

(c) Let f (x) =x+2, g(x) =x -2 and h(x) =3x for $x \in R$,Where R= Set of real numbers. Find [8M]
$(g \circ f)$, $(f \circ g)$,$(f \circ f)$, $(g \circ g)$,$(f \circ h)$,$(h \circ g)$,$(h \circ f)$,$(f \circ h \circ g)$

Q.4 (a) Let R is a binary relation. Let S = {(a, b) | (a, c)∈ R and (c, b) ∈ R for some c} Show that if R is an equivalence relation then S is also an equivalence relation. [6M]

[TURN OVER

(b) Determine the generating function of the numeric function a_r, where [6M]

(i) $a_r = 3^r + 4^{r+1}$, $r \geq 0$
(ii) $a_r = 5$, $r \geq 0$

(c) Consider the (3, 6) encoding function $e: B^3 \rightarrow B^6$ defined by [8M]
e(000)= 000000 e(001)= 001100 e(010)= 010011 e(011)= 011111
e(100)= 100101 e(101)= 101001 e(110)= 110110 e(111)= 111010
Decode the following words relative to a maximum likelihood decoding function.
(i) 000101 (ii) 010101

Q.5 (a) Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. [6M]

(b) Use mathematical induction to show that [6M]
$1+5+9+..........+(4n-3) = n(2n-1)$

(c) Find the greatest lower bound and least upper bound of the set {3, 9, 12} and {1, 2, 4, 5, 10} if they exists in the poset (z+, /).Where / is the relation of divisibility. [8M]

Q.6 (a) Let A= {1,2,3,4}and Let R = {(1,1) (1,2) (1,4) (2,4) (3,1) (3,2) (4,2) (4,3) (4,4)} .Find transitive closure by Warshall's algorithm. [6M]

(b) Let H= $\{[0]_6, [3]_6\}$ find the left and right cosets in group Z_6. Is H a normal subgroup of group of Z_6. [6M]

(c) Find the complete solution of the recurrence relation [8M]
$a_n + 2a_{n-1} = n + 3$ for $n \geq 1$ and with $a_0 = 3$
