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S.E. (Computer Engineering/IT) (II Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Answer Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Your answers will be valued as a whole.

(v) Use of electronic pocket calculator is allowed.

(vi) Assume suitable data, if necessary.

1. (a) Solve any *two* of the following : [8]

(i)  $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$  (use method of variation of parameters)

(ii)  $(D^2 - 4)y = e^{4x} + 2x^3$

(iii)  $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y = 24x.$

P.T.O.

- (b) Solve the following integral equation using Fourier transform : [4]

$$\int_0^{\infty} f(x) \sin \lambda x d\lambda = 1 - \lambda, \quad 0 \leq \lambda \leq 1$$
$$= 0 \quad , \quad \lambda \geq 1$$

Or

2. (a) An electrical circuit consists of an inductance 0.1 henry, a resistance  $R$  of 20 ohms and a condenser of capacitance  $C$  of 25 microfarads. If the differential equation of electric circuit is : [4]

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0,$$

then find the charge  $q$  and current  $i$  at any time  $t$ , given

that when  $t = 0$ ,  $q = 0.05$  coulombs and  $i = \frac{dq}{dt} = 0$ .

- (b) Solve any one : [4]

- (i) Find :

$$z^{-1} \left\{ \frac{1}{(z-4)(z-5)} \right\}$$

by inversion integral method.

- (ii) Find  $z$ -transform of :

$$f(k) = (k+1) a^k, \quad k \geq 0.$$

- (c) Using  $z$  transform, solve the following difference equation : [4]

$$f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k, \quad k \geq 0$$
$$f(0) = 0.$$

3. (a) The first four moments of a distribution about the value 4 of the variable are  $-1.5$ ,  $17$ ,  $-30$  and  $108$ . Find the central moments,  $\beta_1$  and  $\beta_2$ . [4]
- (b) By the method of least squares, find the straight line that best fits the following data : [4]

$x$	$y$
1	14
2	27
3	40
4	55
5	68

- (c) There is a small chance of  $1/1000$  for any computer produced to be defective. Determine in a sample of 2000 computers, the probability : [4]
- (i) no defective and
- (ii) 2 defectives.

Or

4. (a) Team A has a probability of  $\frac{2}{3}$  of winning whenever the team plays a particular game. If team A plays 4 games, find the probability that the team wins : [4]
- (i) exactly two games and
- (ii) at least two games.

- (b) The lifetime of a certain component has a normal distribution with mean of 400 hours and standard deviation of 50 hours. Assuming a normal sample of 1000 components, determine approximately the number of components whose lifetime lies between 340 to 465 hours. Given : [4]

$$Z = 1.2 \text{ Area} = 0.3849$$

$$Z = 1.3 \text{ Area} = 0.4032.$$

- (c) Calculate the coefficient of correlation for the following data : [4]

$x$	$y$
10	18
14	12
18	24
22	6
22	30
30	36

5. (a) Find the directional derivative of a function : [4]

$$\phi = 2x^2 + 3y^2 + z^2 \quad \text{at } (2, 1, 3)$$

in the direction of  $(i + j + k)$ .

- (b) Show that the vector field : [4]

$$\vec{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$$

is irrotational and hence find a scalar potential function  $\phi$  such that  $\vec{F} = \nabla\phi$ .

- (c) Find the work done by a force field : [5]

$$\vec{F} = x^2\mathbf{i} + (x-y)\mathbf{j} + (y+z)\mathbf{k}$$

along a straight line from (0, 0, 0) to (2, 1, 2).

Or

6. (a) Find the directional derivative of : [4]

$$\phi = 4xz^2 - 3x^2y^2z \text{ at } (1, 1, 1)$$

in the direction of a vector  $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ .

- (b) Show that (any one) : [4]

$$(i) \quad \nabla \left( \frac{\vec{a} \cdot \vec{r}}{r^3} \right) = \frac{\vec{a}}{r^3} - \frac{3(\vec{a} \cdot \vec{r})\vec{r}}{r^5}$$

where  $\vec{a}$  is a constant vector.

$$(ii) \quad \nabla^4(r^4) = 120.$$

- (c) Evaluate the integral : [5]

$$\oint_C \vec{F} \cdot d\vec{r}$$

along the curve  $x = y = z = t$  from  $t = 0$  to  $t = 2$  where

$$\vec{F} = (x^2 + yz)\mathbf{i} + (y^2 + zx)\mathbf{j} + (z^2 + xy)\mathbf{k}$$

7. (a) If [4]

$$u = 3x^2y - y^3,$$

find  $v$  such that  $f(z) = u + iv$  is analytic.

- (b) Evaluate : [5]

$$\oint_C \frac{z+4}{(z+1)(z+2)} dz,$$

where  $C$  is the circle  $|z| < 3$ .

- (c) Find the bilinear transformation which maps the points  $(1, i, -1)$  from the  $z$  plane into the points  $(i, 0, -i)$  of the  $w$  plane. [4]

Or

8. (a) If [4]

$$u = 3x^2 - 3y^2 + 2y,$$

find  $v$  such that  $f(z) = u + iv$  is analytic. Determine  $f(z)$  in terms of  $z$ .

- (b) Evaluate : [5]

$$\oint_C \frac{Az^2 + z}{z^2 - 1} dz,$$

where  $C$  is the contour  $|z - 1| = \frac{i}{2}$ .

- (c) Find the map of straight line  $y = x$  under the transformation

$$w = \frac{z-1}{z+1}. \quad [4]$$