Total No of Questions: [8] ***SEAT NO. :***

[Total No. of Pages :03]

S.E. (Computer / Information Technology)

Engineering Mathematics - III

2012 Course

Time: 2Hours ***Max. Marks : 50***

Instructions to the candidates:

1) ***Solve Q1 or Q2, Q3 or Q4, Q5 or Q6 and Q7 or Q8.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right side indicate full marks.***
4) ***Use of Calculator is allowed.***
5) ***Assume Suitable data if necessary***

Q1) a) Solve any **two** of the following [08]

i) $(D^2 + 4)y = \cos 3x \cdot \cos x$

ii) $(D^2 + 6D + 9)y = e^{3x}/x^2$ …. (by variation of parameters method)

iii) $x^3 (d^3y/dx^3) + 2x^2 (d^2y/dx^2) + 2y = 20(x + 1/x)$

b) Obtain f(k) given that, [04]

$f(k+2) + 5f(k+1) + 6f(k) = 0, \quad k \ge 0 \quad f(0) = 0, \; f(1) = 2$

by using Z transform.

OR

Q2) a) An emf E sin (pt) is applied at t = 0 to a circuit containing a condenser 'C' and Inductance 'L' in series. The current 'x' satisfies the equation [04]

$L(dx/dt) + \frac{1}{C}\int x\, dt = E\sin(pt)$

Where $= \frac{-dq}{dt}$. If $p^2 = \frac{1}{LC}$ and initially the current x and charge q is zero then show that current in the circuit at any time t is $\frac{E}{2L}\; t\sin(pt)$.

b) Solve the integral equation [04]

$$\int_0^\infty f(x)\cos\lambda x\, dx = \begin{cases} 1-\lambda & 0 \le \lambda \le 1 \\ 0 & \lambda > 1 \end{cases}$$

And hence show that $\int_0^\infty \frac{\sin^2 z}{z^2}\, dz = \pi/2$

c) Attempt **any one** [04]

i) Find the Z- transform of $f(k) = e^{-2k}\cos(5k+3)$

ii) Find the inverse Z- transform of $\frac{z\,(z+1)}{z^2-2z+1}$, $|z| > 1$

Q3) a) The first four moments of a distribution about 2 are 1, 2.5, 5.5 and 16. Calculate the first four moments about the mean, A. M., S. D., β_1 and β_2 [05]

b) In a certain examination 200 students appeared. Average marks obtained were 50% with standard deviation 5%. How many students do you expect to obtain more than 60% of marks, supposing that the marks are distributed normally ? [04]

(Given z =2 ; A = 0.4772)

c) Find the directional derivatives of: [04]

$\emptyset = xy^2 + yz^2 + zx^2$ at (1,1,1) along the line $2(x-2) = y+1 = z-1$

OR

Q4) a) Calculate the coefficient of correlation for the following data [05]

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

b) Prove the following (**Any one**) [04]

i) $\nabla^4 r^4 = 120$

ii) $\nabla.\left[r\nabla\frac{1}{r^5}\right] = \frac{15}{r^6}$

c) Show that $\bar{F} = (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{k}$ is irrotational. Also find Φ such that $\bar{F} = \nabla\Phi$ [04]

Q5) a) Find the work done in moving a particle along [04]

x=a cosΘ, y= a sinΘ, z=bΘ, from $\Theta = \frac{\pi}{4}$ $\boldsymbol{to}$ $\boldsymbol{\Theta} = \frac{\pi}{2}$ under a field of force given by $\bar{\boldsymbol{F}}$ = -3a $\sin^2\Theta\cos\Theta\,\hat{\boldsymbol{\imath}}$ + a $(2\sin\Theta-3\sin^3\Theta)\hat{\boldsymbol{\jmath}}$ + b $\sin 2\Theta\,\hat{\boldsymbol{k}}$

b) Evaluate $\iint_s (yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}).d\bar{s}$, where s is the curved surface of the cone $x^2 + y^2 = z^2$, $z = 4$ [04]

c) Using Stokes Theorem to evaluate $\int_c (4y\hat{\imath} + 2z\hat{\jmath} + 6y\hat{k}).d\bar{r}$ where C is the curve of intersection of $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$ [04]

OR

Q6) a) A vector field is given by $\bar{F} = (2x - cosy)\hat{\imath} + x(4 + siny)\hat{\jmath}$, evaluate $\int_c \bar{F}.d\bar{r}$, where c is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $z = 0$. [04]

b) Prove that $\iiint_v \frac{1}{r^2} dv = \iint_s \frac{1}{r^2} \bar{r}.d\bar{s}$, where s is closed surface enclosing the volume v. Hence evaluate $\iint_s \frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{r^2}.d\bar{s}$, where s is the surface of the sphere $x^2+y^2+z^2 = a^2$. [04]

c) If $\bar{E} = \nabla\emptyset, and\ \nabla^2\emptyset = -4\pi\rho$, [04]

prove that $\iint_s \bar{E}.d\bar{s} = -4\pi \iiint_v \rho\, dv$

Q7) a) Find the value of p such that the function [04]

$f(x) = r^2\cos 2\Theta + i\, r^2 \sin p\Theta$ becomes analytical function.

b) Evaluate $\oint_c \frac{z^2+cos^2 z}{(z-\frac{\pi}{4})^3} dz$ [05]

where c is a circle $x^2+y^2=1$

c) Find the bilinear transformation which maps 1, *i*, -1 from z plane into *i*, 0, -*i* from the w plane [04]

OR

Q8) a) Determine the analytic function f(z) whose real part is [04]

$U = x^3-3xy^2+3x^2-3y^2+1$

b) Evaluate $\oint_c \left[\frac{sin\pi z^2+2z}{(z-1)^2(z-2)}\right]dz$ [05]

where c is a circle $x^2+y^2=16$

c) Show that the transformation ω = sin z transforms the straight lines x = c of z plane into hyperbolas in the ω plane [04]

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