Total No. of Questions—**8**] [Total No. of Printed Pages—**4+2**

Seat No.	

[4956]-108

F.E. EXAMINATION, 2016

ENGINEERING MATHEMATICS—II

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt *four* questions : Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6,** Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic non-programable calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve the following differential equations : [8]

(*i*) $\frac{dy}{dx} = \frac{2x - 3y + 1}{3x + 4y - 5}$

(*ii*) $x \cos x \frac{dy}{dx} + (\cos x - x \sin x)\, y = 1.$

(*b*) Assuming that the resistance to movement of a ship through water in the form of $(a^2 + b^2 v^2)$, where v is the velocity, a and b are constants, write down the differential equation for retardation of ship moving with engine stopped. Prove that the time in which the speed falls to one half its original value u is given by :

$$\frac{w}{abg} \tan^{-1}\left(\frac{abu}{2a^2 + b^2 u^2}\right)$$

where w is the weigth of the ship. [4]

Or

2. (*a*) Solve : [4]

$$y \log y \; dx + (x - \log y) \; dy = 0$$

(*b*) Solve the following : [8]

(*i*) A body of temperature 80°F is placed in a room of constant temperature 50°F at time $t = 0$. At the end of 5 minutes the body was cooled to a temperature of 70°F. Find the time at which temperature of the body will be 60°F.

(*ii*) A capacitor C = 0.01 F in series with a resistor R = 20 Ω is charged from a battery 10 Volts. Assuming that initially the capacitor is completely uncharged, determine the charge $Q(t)$ and current $I(t)$ in the circuit.

3. (*a*) Find the Fourier series of : [5]

$$f(x) = x^3, \; -\pi < x < \pi.$$

(*b*) Evaluate : [3]

$$\int_0^\infty \frac{x^9\left(1 - x^5\right)}{\left(1 + x\right)^{25}}\,dx.$$

(*c*) Trace the following curve (any *one*) : [4]

(*i*) $x = a(t + \sin t),\ y = a(1 + \cos t)$

(*ii*) $r = a \cos 3\theta.$

Or

4. (*a*) If [4]

$$I_n = \int_0^{\pi/4} \sec^n \theta \, d\theta,$$

prove that :

$$I_n = \frac{\left(\sqrt{2}\right)^{n-2}}{n - 1} + \frac{n + 2}{n - 1} I_{n-2}.$$

(*b*) If [4]

$$f(x) = \int_2^x (x - t)\, G(t)\, dt$$

then show that :

$$\frac{d^2 f}{dx^2} - G(x) = 0.$$

(*c*) Find the length of arc of an Astroid : [4]

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}.$$

5. (a) Show that the plane

$$2x - 2y + z + 12 = 0$$

touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0.$$

Also find the point of contact. [5]

(b) Find the equation of right circular cone passing through (2, –2, 1) with vertex at origin and axis parallel to the line : [4]

$$\frac{x-2}{5} = \frac{y-1}{1} = \frac{z+2}{1}.$$

(c) Find the equation of right circular cylinder whose axis is :

$$x = 2y = -z$$

and radius is 4. [4]

Or

6. (a) Find the equation of the sphere which has its centre at (2, 3, –1) and touches the line : [5]

$$\frac{x+1}{-5} = \frac{y-8}{3} = \frac{z-4}{4}.$$

(b) Find the equation of the cone with vertex at (1, 2, –3), semi-vertical angle $\cos^{-1}\frac{1}{\sqrt{3}}$ and the line :

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+1}{-1}$$

as the axis of the cone. [4]

(*c*) Find the equation of right circular cylcinder of raidus 2 with the axis : [4]

$$\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}.$$

7. Attempt any *two* of the following :

(*a*) Evaluate : [6]

$$\iint \frac{1}{x^4 + y^2}\, dxdy$$

over the region

$$y \geq x^2,\ x \geq 1.$$

(*b*) Evaluate : [7]

$$\iiint \frac{dxdydz}{\sqrt{1 - x^2 - y^2 - z^2}}$$

taken throughout the volume of the sphere :

$$x^2 + y^2 + z^2 = 1$$

in the positive octant.

(*c*) Find the area bounded by the parabola

$$y^2 = 4x$$

and the straight line : [6]

$$2x - 3y + 4 = 0.$$

Or

8. Attempt any *two* of the following :

(*a*) Evaluate : [6]

$$\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log_e\left(x^2 + y^2\right) dxdy.$$

(*b*) A rod of length l is divided into two parts at random. Find average of sum of squares of these parts. Also find mean value of rectangle contained by these two segments. [7]

(*c*) Find the volume common to the cylinders : [6]

$$x^2 + y^2 = a^2 \text{ and}$$
$$x^2 + z^2 = a^2.$$