Total No. of Questions : 8]

SEAT No. :

P4417

[Total No. of Pages : 3

[5251]-1001
F.E.
ENGINEERING MATHEMATICS - I
(2015 Pattern)

Time : 2 Hours] *[Max. Marks : 50*

Instructions to the candidates :

1) *Solve Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7, or Q.8.*
2) *Neat diagrams must be drawn, wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Use of electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

Q1) a) Reduce the following matrix to its normal form and hence find the rank. **[4]**

$$A=\begin{bmatrix}1 & 1 & -1 & 1\\ 1 & -1 & 2 & -1\\ 3 & 1 & 0 & 1\end{bmatrix}$$

b) Show that $A=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$ is an orthogonal matrix. **[4]**

c) A square lies above real axis in argand diagram, and of its adjecent vertices are the origin and the point 5 + 6i, find the complex numbers representing other vertices. **[4]**

OR

Q2) a) Verify Cayley-Hamilton theorem for

$$A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix} \text{ find } A^{-1}$$ **[4]**

b) If $\tan(x+iy)=i$, where x and y are real, prove that x is indeterminate and y is infinite. **[4]**

c) Considering the principal value, express in the form a + ib the expression. $\left(\sqrt{i}\right)^{\sqrt{i}}$. **[4]**

P.T.O.

Q3) a) Test the convergence of the series (any one) **[4]**

i) $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+------+\frac{n}{1+2^n}+-----$

ii) $\sum_{n=1}^{\infty}\frac{10n+4}{n^3}$

b) Expand $(1+x)^{1/x}$ in ascending powers of x, expansion being correct upto second power of *x*. **[4]**

c) Find n^{th} derivative of $y=\frac{2x+3}{(x-1)(x-2)}$ **[4]**

OR

Q4) a) Solve any one **[4]**

i) $\lim_{x\to\frac{1}{2}}\frac{\cos^2\pi x}{e^{2x}-2xe}$

ii) $\lim_{x\to 1}(1-x^2)^{\frac{1}{\log(1-x)}}$

b) Using Taylor's theorem, express $5 + 4(x-1)^2 - 3(x-1)^3 + (x-1)^4$ in ascending powers of x. **[4]**

c) If $y=e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$ **[4]**

Q5) Solve any two

a) If $z^3-zx-y=4$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ **[6]**

b) If $u=\frac{xyz}{2x+y+z}+\log\left(\frac{x^2+y^2+z^2}{xy+yz}\right)$ Find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$ **[7]**

c) If $x = u + v + w$, $y = uv + vw + uw$, $z = uvw$ and ϕ is a function of x,y,z then prove that $u\frac{\partial\phi}{\partial u}+v\frac{\partial\phi}{\partial v}+w\frac{\partial\phi}{\partial w}=x\frac{\partial\phi}{\partial x}+2y\frac{\partial\phi}{\partial y}+3z\frac{\partial\phi}{\partial z}$ **[6]**

OR

Q6) Solve any two

a) Find $\frac{dz}{dx}$ if $z = x^2y$ and $x^2 + xy + y^2 = 1$ **[6]**

b) If $u = \operatorname{cosec}^{-1}\sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}$

Prove that $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = \frac{\tan u}{144}[\tan^2 u + 13]$. **[7]**

c) If $x = \frac{r}{2}\left[e^{\theta} + e^{-\theta}\right]$ and $y = \frac{r}{2}\left[e^{\theta} + e^{-\theta}\right]$ prove that $\left(\frac{\partial x}{\partial r}\right)_{\theta} = \left(\frac{\partial r}{\partial x}\right)_{y}$ **[6]**

Q7) a) If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ **[4]**

b) Examine for functional dependence **[4]**

$u = \sin^{-1}x - \sin^{-1}y,\ v = x\sqrt{1-y^2} - y\sqrt{1-x^2}$

c) Find the extreme values of the function $f(x,y) = x^2 + y^2 + 6x + 12$. **[5]**

OR

Q8) a) If $ux + vy = 0,\ \frac{u}{x} + \frac{v}{y} = 1$ then using Jacobian find $\left(\frac{\partial u}{\partial x}\right)_y$. **[4]**

b) The focal length of a mirror is found from the formula : $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$.

Find the percentage error in f if u and v are both in error by p% each. **[4]**

c) Find the point on the surface $z^2 = xy + 1$ nearest to the origin, by using lagranges method. **[5]**