Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

Seat No.	

[4957]-1050

S.E. (Electronics/E&TC) (Second Semester) EXAMINATION, 2016
ENGINEERING MATHEMATICS-III
(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of logarithmic tables, electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve (any *two*) : [8]

(*i*) $(D^3 + 4D)\, y = \sin 5x \cos 3x$

(*ii*) $(D^2 + 2D + 1)y = \dfrac{e^{-x}}{x+2}$

(*iii*) $(x+2)^2 \dfrac{d^2y}{dx^2} + 3(x+2)\dfrac{dy}{dx} + y = \cos \log (x+2)$

(*b*) Find Fourier sine transform of : [4]

$$f(x) = \frac{e^{-ax}}{x}$$

Or

2. (*a*) An e.m.f. 200 V is in series with a 10 Ω resistor, a 1 henry inductor and 0.02 farad capacitor. At $t = 0$, Q = I = 0. Find charge Q and current, I at any time t. [4]

(*b*) Solve (any *one*) : [4]

(*i*) Find z-transform of :

$$f(k) = k^2 \; e^{-3k}, \; k \ge 0$$

(*ii*) Find inverse z-transform of :

$$F(z) = \frac{z^3}{(z-1)(z-2)^2}, |\,z| > 2$$

(*c*) Solve : [4]

$$f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k, \; k \ge 0, f(0) = 0$$

3. (*a*) Using fourth order Runge-Kutta method, solve the differential equation : [4]

$$\frac{dy}{dx} = \sqrt{x+y}$$

with $y(0) = 1$, and find $y(0.2)$ taking $h = 0.2$.

(*b*) Find Lagrange's interpolating polynomial passing through set of points : [4]

x	*y*
0	4
1	3
2	6

(*c*) Find the directional derivative of :

$$\phi = e^{2x} \cos yz$$

at the origin in the direction tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \pi/4$. [4]

Or

4. (*a*) With usual notations prove (any *one*) : [4]

(*i*) $\nabla \times (\bar{a} \times \bar{r}) = 2\bar{a}$, $\bar{a}$ is constant vector

(*ii*) $\nabla^2 \left(\dfrac{\bar{a} \cdot \bar{b}}{r} \right) = 0$

(*b*) Show that :

$$\overline{\mathrm{F}} = \frac{xi + yj}{x^2 + y^2}$$

is solenoidal as well as irrotational. [4]

(*c*) Evaluate : [4]

$$\int_0^1 \frac{1}{1 + x} \, dx$$

by Simpson's $\left(\frac{3}{8}\right)$th rule.

5. (*a*) If : [4]

$$\overline{\mathrm{F}} = (2x + y^2)\,\bar{i} + (3y - 4x)\,\bar{j},$$

then evaluate $\int_{\mathrm{C}} \overline{\mathrm{F}} \, d\bar{r}$ around the parabolic curve $y = x^2$ joining (0, 0) and (1, 1) [4]

(*b*) Show that : [4]

$$\iint \frac{\bar{r}}{r^3} \cdot \hat{n} \, d\mathrm{S} = 0$$

(c) Evaluate :

$$\iint (\nabla \times \overline{F}) \, . \, \hat{n} \, dS$$

where 'S' is the curved surface of the paraboloid :

$$x^2 + y^2 = 2z$$

bounded by the plane $z = 2$, where : [5]

$$\overline{F} = 3(x - y)\, \overline{i} + 2xz\, \overline{j} + xy\, \overline{k}$$

Or

6. (a) Using Green's theorem show that : [4]

$$\int_C \overline{F} \, . \, d\overline{r} = 0$$

where :

$$\overline{F} = x\overline{i} + y^2\, \overline{j},$$

over the first quadrant of the circle :

$$x^2 + y^2 = a^2.$$

(b) If [4]

$$\overline{E} = \nabla\phi \text{ and } \nabla^2\phi = -4\, p\pi,$$

prove that :

$$\iint_s \overline{E} \, . \, d\overline{s} = -4\, p\pi \iiint_v dv$$

(c) Evaluate : [5]

$$\iint_s \text{curl } \overline{F} \, . \, \hat{n} \, ds$$

for the surface of paraboloid

$z = 9 - (x^2 + y^2)$, where $\overline{F} = (x^2 + y - 4)\, \overline{i} + 3xy\, \overline{j} + (2xz + z^2)\overline{k}$

7. (a) Obtain the analytic function $f(z)$ such that : [4]

$$\text{Im}\{f(z)\} = \tan^{-1} y/x \text{ and } f(1) = 0$$

(b) Evaluate : [4]

$$\int_C \frac{z^4}{z - 3i}\, dz$$

where :

$$C = \{z \mid |z - 2| < 5\}.$$

(c) Find the bilinear transformation which sends the points 1, i, -1 from z-plane into the points i, 0, $-i$ of the w-plane. [5]

Or

8. (a) If $f(z)$ is an analytic function with constant modulus then prove that $f(z)$ is a constant function. [4]

(b) Evaluate : [4]

$$\int_C \frac{z\, dz}{(z-1)(z-2)^2},$$

where C is the circle $|z - 2| = \frac{1}{2}$.

(c) Show that under the transformation $w = z + \frac{4}{z}$ the circle $|z| = 3$ is mapped onto the ellipse. [5]