Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

Seat No.	

[4857]-1050

S.E. (Electronics/E&TC) (II Sem.) EXAMINATION, 2015
ENGINEERING MATHEMATICS—III
(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable pocket calculator (electronic) is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Attempt and solve any *two* : [8]

(*i*) $\left(D^2 + 13D + 36\right)y = e^{-4x} + \sinh x$

(*ii*) $x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^2 + 2\log x$

(*iii*) $\dfrac{d^2y}{dx^2} + y = \sec x \tan x$ (by variation of parameters).

(*b*) Find the Fourier cosine integral representation of the function : [4]

$$f(x) = \begin{cases} x & 0 \le x \le a \\ 0 & x > a \end{cases}$$

Or

2. (*a*) An inductor of 0.5 henries is connected in series with a resistor of 6 ohms, a capacitor 0.02 farads, a generator having alternative voltage as $24 \sin 10t$, $t > 0$. Find the charge and current at time 't'. [4]

(*b*) Attempt any *one* : [4]

(*i*) Find z transform of $\left(\frac{2}{3}\right)^{|k|}$ for all k.

(*ii*) Find z inverse of $\frac{3z^2 + 2z}{z^2 - 3z + 2}$, $1 < |z| < 2$.

(*c*) Solve :

$$f(k + 2) + 6f(k + 1) + 9f(k) = 2^k$$

given $f(0) = f(1) = 0$. [4]

3. (*a*) Solve the following differential equation $\frac{dy}{dx} = x - 2y$, using Runge-Kutta fourth order method, given that $y = 1$ when $x = 0$ and find y at $x = 0.1$. [4]

(*b*) Find Lagrange's Interpolating Polynomial satisfying the data : [4]

x	y
0	2
1	3
2	12
5	147

(*c*) Find the directional derivative of the function $\phi = x^2y + xyz + z^3$, at (1, 2, –1) along the normal to the surface $x^2y^3 = 4xy + y^2z$ at (1, 2, 0). [4]

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla \times (\bar{r} \times \bar{u}) = \bar{r}(\nabla \cdot \bar{u}) - (\bar{r} \cdot \nabla)\bar{u} - 2\bar{u}$

(*ii*) $\nabla^4 e^r = \left(1 + \frac{4}{r}\right)e^r.$

(*b*) Show that :

$$\bar{F} = \frac{1}{r}\left[r^2\bar{a} + (\bar{a} \cdot \bar{r})\bar{r}\right]$$

is irrotational where $\bar{a}$ is a constant vector. [4]

(*c*) Evaluate :

$$\int_1^2 \frac{dx}{x},$$

using Simpson's $\left(\frac{1}{3}\right)$rd rule, taking $h = 0.25$. [4]

5. (*a*) Evaluate :

$$\int_C \bar{F} \cdot d\bar{r},$$

where

$$\bar{F} = (3x^2 - 6yz)i + (2y + 3xz)j + (1 - 4xyz^2)k$$

along a line joining (0, 0, 0) and (1, 2, 3). [4]

(*b*) Use Green's Lemma to evaluate :

$$\int_C \left(xy - x^2\right)dx + x^2y\,dy,$$

over the region bounded by the curves $y = x$, $y = 0$, $x = 1$. [4]

(*c*) Use Stokes' theorem to evaluate :

$$\oint \overline{F} \cdot d\overline{r},$$

where

$$\overline{F} = xyi + yzj + z^2k,$$

over a cube having open base and length of side of cube is 'a' unit. [5]

Or

6. (*a*) Find the work done by a force field :

$$\overline{F} = 3x^2i + (2xz - y)j + zk,$$

along the path $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. [4]

(*b*) Use Gauss Divergence theorem to evaluate :

$$\iint_S \left(\overline{F} \cdot \hat{n}\right)ds,$$

where

$$\overline{F} = \left(x + y^2\right)i - 2xj + 2yzk,$$

where s is a surface of tetrahedron bounded by co-ordinate planes and $2x + y + 2z = 6$. [5]

(c) If $\nabla \cdot \bar{B} = 0$, $\bar{B} = \nabla \times \bar{A}$, $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$, prove that :

$$\bar{E} + \frac{\partial \bar{A}}{\partial t} = \text{grad } v,$$

where v is some scalar point function. [4]

7. (a) If $u = 4xy\left(x^2 - y^2\right)$ find its harmonic conjugate v. [4]

(b) Evaluate :

$$\int_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$$

where C is $|z| = 1$. [5]

(c) Find bilinear transformation which maps 0, –1, i on to the points 2, ∞, $\frac{1}{2}(5 + i)$. [4]

Or

8. (a) Show that the transformation $\omega = \frac{z - a}{z + a}$ maps the right half of the z-plane into the unit circle $|\omega| < 1$. [4]

(b) If

$$f(a) = \int_C \frac{3z^2 + 5z + 2}{z - a} dz$$

where C is ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ find $f(1)$. [4]

(c) If $f(z)$ is an analytic function of z, $f(z) = u + iv$, prove that : [5]

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left|\text{Re} f(z)\right|^2 = 2\left|f'(z)\right|^2.$$