USN | | | | | | | | | | |     **10MAT41**

# Fourth Semester B.E. Degree Examination, June/July 2016
## Engineering Mathematics – IV

Time: 3 hrs.     Max. Marks:100

**Note:** ***1. Answer any FIVE full questions, selecting atleast TWO questions from each part.***
***2. Use of statistical tables permitted.***



## PART – A

1 a. Using Taylor's series method, solve $y' = x + y^2$, $y(0) = 1$ at $x = 0.1, 0.2$, considering upto 4th degree term. **(06 Marks)**

b. Using modified Euler's method, find an approximate value of y when x = 0.2 given that $\frac{dy}{dx} = x + y$ and y = 1 when x = 0. Take h = 0.1. Perform two iterations in each stage. **(07 Marks)**

c. Using Adams-Bashforth method, obtain the solution of $\frac{dy}{dx} = x - y^2$ at x = 0.8 given that y(0) = 0, y(0.2) = 0.0200, y(0.4) = 0.0795, y(0.6) = 0.1762. Apply the corrector formula twice. **(07 Marks)**



2 a. Employing the Picard's method, obtain the second order approximate solution of the following problem at x = 0.2, $\frac{dy}{dx} = x + yz$, $\frac{dz}{dx} = y + zx$, $y(0) = 1$, $z(0) = -1$. **(06 Marks)**

b. Solve $\frac{dy}{dx} = 1 + xz$ and $\frac{dz}{dx} = -xy$ for x = 0.3 by applying Runge Kutta method given y(0) = 0 and z(0) = 1. Take h = 0.3. **(07 Marks)**

c. Using the Milne's method, obtain an approximate solution at the point x = 0.4 of the problem $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$ given that y(0) = 1, y(0.1) = 1.03995, y(0.2) = 1.138036, y(0.3) = 1.29865, y′(0) = 0.1, y′(0.1) = 0.6955, y′(0.2) = 1.258, y′(0.3) = 1.873. **(07 Marks)**

3 a. Define an analytic function and obtain Cauchy-Riemann equations in polar form. **(06 Marks)**

b. Show that $u = e^{2x}(x\cos 2y - y\sin 2y)$ is a harmonic function and determine the corresponding analytic function. **(07 Marks)**

c. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(07 Marks)**

4 a. Evaluate using Cauchy's integral formula $\int_c \frac{\cos \pi z}{z^2 - 1} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$. **(06 Marks)**

b. Find the bilinear transformation which maps 1, i, -1 to 2, i, -2 respectively. Also find the fixed points of the transformation. **(07 Marks)**

c. Discuss the conformal transformation of $w = z^2$. **(07 Marks)**

## PART – B

5 a. Reduce the differential equation:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(k^2x^2-n^2)y=0$$ into Bessel form and write the complete solution in terms of $\tau_n(x)$ and $\tau_{-n}(x)$. **(06 Marks)**

b. Express $f(x)=x^3+2x^2-x-3$ in terms of Legendre polynomials. **(07 Marks)**

c. If $\alpha$ and $\beta$ are the roots of $\tau_n(x)=0$ then prove that

$$\int_0^1 x\tau_n(\alpha x)\tau_n(\beta x)\,dx=\begin{cases}0, & \alpha\neq\beta\\ \frac{1}{2}[\tau_{n+1}(\alpha)]^2, & \alpha=\beta\end{cases}.$$ **(07 Marks)**

6 a. The probability that sushil will solve a problem is 1/4 and the probability that Ram will solve it is 2/3. If sushil and Ram work independently, what is the probability that the problem will be solved by (i) both of them; (ii) at least one of them? **(06 Marks)**

b. A committee consists of 9 students two of which are from first year, three from second year and four from third year. Three students are to be removed at random. What is the chance that (i) the three students belong to different classes; (ii) two belong to the same class and third to the different class; (iii) the three belong to the same class? **(07 Marks)**

c. The contents of three urns are: 1 white, 2 red, 3 green balls, 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. Two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn. **(07 Marks)**



7 a. The probability mass function of a variate X is

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p(x) | k | 3k | 5k | 7k | 9k | 11k | 13k |

i) Find k

ii) Find $p(x<4)$, $p(x\geq 5)$, $p(3<x\leq 6)$, $p(x>1)$

iii) Find the mean. **(06 Marks)**

b. Derive the mean and variance of Poisson distribution. **(07 Marks)**

c. The mean height of 500 students is 151cm and the standard deviation is 15cm. Assuming that the heights are normally distributed, find how many students heights i) lie between 120 and 155cm; ii) more than 155cm. [Given A(2.07) = 0.4808 and A (0.27) = 0.1064, where A(z) is the area under the standard normal curve from 0 to $z>0$]. **(07 Marks)**

8 a. The means of simple samples of sizes 1000 and 2000 are 67.5 and 68.0cm respectively. Can the samples be regarded as drawn from the same population of S.D 2.5cm [Given $z_{0.05}=1.96$]. **(06 Marks)**

b. A random sample of 10 boys had the following I.Q: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q of 100? [Given $t_{0.05}$ for 9d.f = 2.26]. **(07 Marks)**

c. The following table gives the number of aircraft accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week.

| Days : | Sun | Mon | Tue | Wed | Thur | Fri | Sat | Total |
|---|---|---|---|---|---|---|---|---|
| No. of accidents : | 14 | 16 | 8 | 12 | 11 | 9 | 14 | 84 |

[Given $\psi^2_{0.05}$ 6d.f = 12.59] **(07 Marks)**

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