

CBCS Scheme

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15MAT41

**Fourth Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics-IV**

Time: 3 hrs.

Max. Marks: 80

**Note: 1. Answer FIVE full questions, choosing one full question from each module.
2. Use of statistical tables are permitted.**

Module-1

1 a. Find by Taylor's series method the value of y at $x = 0.1$ from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ (upto 4th degree term). (05 Marks)

b. The following table gives the solution of $5xy' + y^2 - 2 = 0$. Find the value of y at $x = 4.5$ using Milne's predictor and corrector formulae. (05 Marks)

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

c. Using Euler's modified method. Obtain a solution of the equation $\frac{dy}{dx} = x + \sqrt{y}$, with initial conditions $y = 1$ at $x = 0$, for the range $0 \leq x \leq 0.4$ in steps of 0.2. (06 Marks)

OR

2 a. Using modified Euler's method find $y(20.2)$ and $y(20.4)$ given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with $y(20) = 5$ taking $h = 0.2$. (05 Marks)

b. Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Evaluate $y(1.4)$ by Adams-Bashforth method. (05 Marks)

c. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$ by taking $h = 0.2$. (06 Marks)

Module-2

3 a. Obtain the solution of the equation $2\frac{d^2y}{dx^2} = ux + \frac{dy}{dx}$ by computing the value of the dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following data: (05 Marks)

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

b. Express $f(x) = 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (05 Marks)

c. Obtain the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 + n^2)y = 0$ (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8 = 50, will be treated as malpractice.

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OR

4 a. By Runge-Kutta method solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for $x = 0.2$. Correct to four decimal places using the initial conditions $y = 1$ and $y' = 0$ at $x = 0, h = 0.2$. (05 Marks)

b. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ (05 Marks)

c. Prove the Rodrigues formula,

$$\rho_n(x) = \frac{1}{2^n n!} \frac{d^n(x^2 - 1)^n}{dx^n}$$
 (06 Marks)

Module-3

5 a. State and prove Cauchy's-Riemann equation in polar form. (05 Marks)

b. Discuss the transformation $W = e^z$. (05 Marks)

c. Evaluate $\int_C \left\{ \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)^2(z-2)} \right\} dz$ using Cauchy's residue theorem where 'C' is the circle $|z| = 3$ (06 Marks)

OR

6 a. Find the analytic function whose real part is, $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. (05 Marks)

b. State and prove Cauchy's integral formula. (05 Marks)

c. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = -1, -i, 1$. Also find the fixed points of the transformation. (06 Marks)

Module-4

7 a. Find the mean and standard deviation of Poisson distribution. (05 Marks)

b. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for,

- (i) more than 2150 hours.
 - (ii) less than 1950 hours
 - (iii) more than 1920 hours and less than 2160 hours.
- [A(1.833) = 0.4664, A(1.5) = 0.4332, A(2) = 0.4772]

(05 Marks)

c. The joint probability distribution of two random variables x and y is as follows:

x/y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine:

- (i) Marginal distribution of x and y.
- (ii) Covariance of x and y
- (iii) Correlaiton of x and y.

(06 Marks)

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OR

- 8 a. The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured what is the probability that, (i) Exactly 2 are defective (ii) at least 2 are defective (iii) none of them are defective. (05 Marks)
- b. Derive the expressions for mean and variance of binomial distribution. (05 Marks)
- c. A random variable X take the values -3, -2, -1, 0, 1, 2, 3 such that $P(x = 0) = P(x < 0)$ and $P(x = -3) = P(x = -2) = P(x = -1) = P(x = 1) = P(x = 2) = P(x = 3)$. Find the probability distribution. (06 Marks)

Module-5

- 9 a. In 324 throws of a six faced 'die' an odd number turned up 181 times. Is it reasonable to think that the 'die' is an unbiased one? (05 Marks)
- b. Two horses A and B were tested according to the time (in seconds) to run a particular race with the following results:

Horse A:	28	30	32	33	33	29	34
Horse B:	29	30	30	24	27	29	

Test whether you can discriminate between the two horses. ($t_{0.05} = 2.2$ and $t_{0.02} = 2.72$ for 11 d.f) (05 Marks)

- c. Find the unique fixed probability vector for the regular stochastic matrix, $A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ (06 Marks)

OR

- 10 a. Define the terms: (i) Null hypothesis (ii) Type-I and Type-II error (iii) Confidence limits. (05 Marks)

- b. Prove that the Markov chain whose t.p.m $P = \begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is irreducible. Find the corresponding stationary probability vector. (05 Marks)

- c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball. (ii) B has the ball. (iii) C has the ball. (06 Marks)

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