Total No. of Questions—**8**] [Total No. of Printed Pages—**4+2**

Seat No.	

[4657]-571

S.E. (Computer Engineering/Information Technology)

(II Sem.) EXAMINATION, 2014

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt **4** questions : Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic non-programmable calculator is allowed.

(*v*) Assume suitable data whenever necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $\left(D^2 + 6D + 9\right)y = x^{-3}e^{-3x}$

(*ii*) $\left(D^2 - 2D + 2\right)y = e^x \tan x$ (by variation of parameters method)

(*iii*) $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = x^2 \sin\left(\log x\right).$

(*b*) Find the Fourier sine and cosine transforms of e^{-mx}, $m > 0$. [4]

P.T.O.

Or

2. (*a*) The currents x and y in the coupled circuits are given by :

$$(LD + 2R)x - Ry = E$$

$$(LD + 2R)y - Rx = 0.$$

Find the general values of x and y in terms of t. [4]

(*b*) Find the inverse z-transform (any *one*) : [4]

(*i*) $F(z) = \dfrac{10z}{(z-1)(z-2)}$ (by inversion integral method)

(*ii*) $F(z) = \dfrac{z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}, \ |z| > \dfrac{1}{4}.$

(*c*) Solve the difference equation : [4]

$$f(K + 1) - f(K) = 1, \ K \geq 0, \ f(0) = 0.$$

3. (*a*) The first four moments about 44.5 of a distribution are –0.4, 2.99, –0.08 and 27.63. Calculate moments about mean, coefficients of Skewness and Kurtosis. [4]

(*b*) The incidence of a certain disease is such that on the average 20% of workers suffer from it. If 10 workers are selected at random, find the probability that : [4]

(*i*) exactly 2 workers suffer from disease.

(*ii*) not more than 2 workers suffer.

(*c*) Find the directional derivative of :

$$\phi = 4xz^3 - 3x^2y^2z$$

at (2, –1, 2) along a line equally inclined with coordinate axes. [4]

Or

4. (*a*) A random sample of 200 screws is drawn from a population which represents size of screws. If a sample is normally distributed with a mean 3.15 cm and S.D. 0.025 cm, find expected number of screws whose size falls between 3.12 cm and 3.2 cm. [4]

[Given : For $z = 1.2$, area = 0.3849; for $z = 2$, area = 0.4772]

(*b*) Show that (any *one*) : [4]

(*i*) $\nabla \cdot \left(\frac{\bar{a} \times \bar{r}}{r} \right) = 0$

(*ii*) $\nabla^4 \left(r^2 \log r \right) = \frac{6}{r^2}$.

(*c*) A fluid motion is given by :

$$\bar{v} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + \left(xy \cos z + y^2\right)\hat{k}.$$

Is the motion irrotational ? If so, find the scalar velocity potential. [4]

5. (*a*) Find the work done by the force :

$$\bar{F} = \left(x^2 - yz\right)i + \left(y^2 - zx\right)j + \left(z^2 - xy\right)k$$

in taking a particle from (1, 1, 1) to (3, –5, 7). [4]

(*b*) Use divergence theorem to evaluate :

$$\iint_S \left(y^2z^2\ i + z^2x^2\ j + x^2y^2\ k\right)\cdot d\bar{s}$$

where s is the upper half of the sphere $x^2 + y^2 + z^2 = 9$ above the $x\,o\,y$ plane. [5]

(*c*) Apply Stokes' theorem to evaluate :

$$\int_C (4y\ dx + 2z\ dy + 6y\ dz)$$

where C is the curve $x^2 + y^2 + z^2 = 6z,\ z = x + 3.$ [4]

Or

6. (*a*) Find the work done in moving a particle from (0, 1, –1) to $\left(\frac{\pi}{2}, -1, 2\right)$ in a force field : [4]

$$\bar{F} = \left(y^2\cos x + z^3\right)i + (2y\sin x - 4)\,j + \left(3xz^2 + 2\right)k.$$

(*b*) Evaluate :

$$\iint_S \left[\left(x + y^2\right)i - 2x\,j + 2yz\,k\right]\cdot d\bar{s}$$

where s is the plane $2x + y + 2z - 6 = 0$ considered as one of the bounding planes of the tetrahedron $x = 0$, $y = 0$, $z = 0$, $2x + y + 2z = 6$. [5]

(*c*) Verify Stokes' theorem for :

$$\bar{F} = -y^3 i + x^3 j$$

and the closed curve c is the boundary of the circle $x^2 + y^2 = 1$. [4]

7. (*a*) Find the condition under which :

$$u = ax^3 + bx^2y + cxy^2 + dy^3$$

is harmonic. [4]

(*b*) Evaluate :

$$\oint_C \frac{4z^2 + z}{z^2 - 1} dz,$$

where C : $|z - 1| = 3$. [5]

(*c*) Show that :

$$w = \frac{z - i}{1 - iz}$$

maps upper half of z-plane onto interior of unit circle in w-plane. [4]

Or

8. (*a*) Find the harmonic conjugate of : [4]

$$u = r^3 \cos 3\theta + r \sin \theta.$$

(*b*) Evaluate :

$$\oint_C \frac{\sin 2z}{\left(z + \frac{\pi}{3}\right)^4}\, dz,$$

where C : $|z| = 2$. [5]

(*c*) Find the bilinear transformation which maps the points 1, 0, i of the z-plane onto the points ∞, -2, $-\frac{1}{2}(1 + i)$ of the w-plane. [4]