

(3hours)

[Total marks: 80]

- N.B.** (1) Question No. 1 is compulsory.
 (2) Answer **any Three** from remaining
 (3) Figures to the right indicate full marks.

1. (a) Find Laplace transform of $e^{-4t} \sin ht \sin t$. 5

(b) Does there exist an analytic function whose real part is $x^3 - 3x^2y - y^3$. Give justification. 5

(c) Show that $\{\cos x, \cos 2x, \cos 3x, \dots\}$ is a set of orthogonal functions over an interval $(-\pi, \pi)$. 5

(d) Evaluate $\int_0^{2+i} z^2 dz$ along the line joining the point $z_1 = 0$ and $z_2 = 2 + i$. 5

2. (a) Obtain the Taylor's and Laurent series which represent the function,

$$f(z) = \frac{1}{(z+1)(z+3)} \text{ valid in the regions,}$$

(i) $|z| < 1$ (ii) $1 < |z| < 3$ (iii) $|z| > 3$ 6

(b) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$. 6

(c) Using Laplace transform, solve the differential equation :

$$\frac{d^2x}{dt^2} + 4x = t \text{ with } x(0) = 1, \quad x'(0) = -2 \quad \text{8}$$

3. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method, given

$$u(0, t) = 0, u(x, 0) = x(4 - x), u(4, t) = 0, \text{ assuming } h = 1, \text{ find } u \text{ upto } t=5. \quad 6$$

(b) Using convolution theorem find the inverse Laplace transform of

$$\frac{s}{(s^2 + 1)(s^2 + 4)}. \quad 6$$

(c) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under boundary condition $u(0, t) = u(l, t) = 0$, $u(x, 0) = x$, l being the length of rod. 8

[TURN OVER]

4. (a) Using Residue theorem, evaluate, $\int_0^{2\pi} \frac{d\theta}{5 + 3\sin \theta}$. 6

(b) Find the inverse Laplace transform of the following:

$$\frac{s^3 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$
 6

(c) Obtain Half Range Sine Series of $f(x) = x(\pi - x)$ in $(0, \pi)$.

Hence, evaluate $-\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2}$.

8

5. (a) If $f(x) = e^{-3x}$, $-1 < x < 1$. Obtain Complex form of $f(x)$ in $(-1, 1)$. 6

(b) Find the orthogonal trajectory of the family of curves $3x^2y - y^3 = c$. 6

(c) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$u(0, t) = 0$, $u(1, t) = 2t$, $u(x, 0) = 0$, for two time steps taking $h = 0.25$. 8

$u(x, 0) = 0$

6. (a) Obtain the Fourier series for $f(x)$ where

$$f(x) = x + \frac{\pi}{2} \quad -\pi < x < 0$$

$$= \frac{\pi}{2} - x \quad 0 < x < \pi$$

6

(b) Prove that $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$ 6

(c) Find bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -1$. Hence, find the image of $|z| \leq 1$ onto the w -plane. 8