

Q.P. Code :23022

[Time: Three Hours]

[ Marks:80]

Please check whether you have got the right question paper.

- N.B:
1. Question.No.1 is compulsory.
  2. Attempt any three questions from Q.2 to Q.6
  3. Use of statistical table permitted.
  4. Figures to the right indicate full marks.

- Q.1
- a) Evaluate  $\int_C \log z \, dz$  where C is the unit circle in the z - plane. 05
- b) Find the eigen values of the adjoint of  $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$  05
- c) If the arithmetic mean of regression coefficient is p and their difference is 2q, find the correlation coefficient. 05
- d) Write the dual of the following L.P.P.  
 Maximise  $Z = 2x_1 - x_2 + 4x_3$   
 Subject to  $x_1 + 2x_2 - x_3 \leq 5$   
 $2x_1 - x_2 + x_3 \leq 6$   
 $x_1 + x_2 + 3x_3 \leq 10$   
 $4x_1 + x_3 \leq 12$   
 $x_1, x_2, x_3 \geq 0$  05
- Q.2
- a) Evaluate  $\int_C \frac{\cot z}{z} \, dz$  where C is the ellipse  $9x^2 + 4y^2 = 1$  06
- b) Show that  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$  is non- derogatory. 06
- c) If X is a normal variate with mean 10 and standard deviation 4, find  
 i)  $P(|X - 14| < 1)$ , ii)  $P(5 \leq X \leq 18)$ , iii)  $P(X \leq 12)$  08

- Q.3
- a) Find the expectation of number of failures preceding the first success in an infinite series of independent trials with constant probabilities  $p$  &  $q$  of success and failure respectively. 06
- b) Using Simplex Method solve the following L.P.P  
 Maximise  $Z = 10x_1 + x_2 + x_3$  06  
 Subject to  $x_1 + x_2 - 3x_3 \leq 10$   
 $4x_1 + x_2 + x_3 \leq 20$   
 $x_1, x_2, x_3 \geq 0$
- c) Expand  $f(z) = \frac{1}{z(z+1)(z-2)}$   
 (i) Within the unit circle about the origin. 08  
 (ii) with in the annulus region between the concentric circles about the origin having radii 1 and 2 respectively.  
 (iii) In the exterior of the circle with centre at the origin and radius 2.
- Q.4
- a) If  $X$  is Binomial distributed with mean=2 and variance =  $4/3$ , find the probability distribution of  $X$ . 06
- b) Calculate the value of rank correlation coefficient from the following data regarding score of 6 students in physics & chemistry test. 06  
 Marks in Physics : 40, 42, 45, 35, 36, 39  
 Marks in Chemistry : 46, 43, 44, 39, 40, 43
- c) Is the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  diagonalisable? If so find the diagonal form and the transforming matrix. 08
- Q.5
- a) A random sample of 50 items gives the mean 6.2 and standard deviation 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance? 06
- b) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2}$ ,  $a>0$  Using Cauchy's residue theorem. 06
- c) Using Kuhn-Tucker condition to solve the following N.L.P.P 08  
 Maximise  $Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$   
 Subject to  $3x_1 + 2x_2 \leq 6$   
 $x_1, x_2 \geq 0$

- Q.6 a) The following table gives the number of accidents in a city during a week. Find whether the accidents are uniformly distributed over a week. 06

Day:	Sun,	Mon,	Tue,	Wed,	Thu,	Fri,	Sat,	Total.
No. of accidents:	13	15	9	11	12	10	14	84

- b) If two independent random samples of sizes 15 & 8 have respectively the following means and population standard deviations, 06

$$\bar{X}_1 = 980 \quad \bar{X}_2 = 1012$$

$$\sigma_1 = 75 \quad \sigma_2 = 80$$

Test the hypothesis that  $\mu_1 = \mu_2$  at 5% level of significance,

(Assume the population to be normal)

08

- c) Using Penally (Big M) method solve the following L.P.P.

$$\text{Minimise } Z = 2x_1 + x_2$$

$$\text{Subject to } 3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \geq 0$$