

Reg. No. :

**Question Paper Code : 80603**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

First Semester

Mechanical Engineering

MA 6151 — MATHEMATICS - I

(Common to all branches except Marine Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the sum of two eigenvalues and trace of a matrix  $A$  are equal, find the value of  $|A|$ .
2. Write down the matrix corresponding to the quadratic form  $2x_1^2 + 5x_2^2 + 4x_1x_2 + 2x_3x_1$ .
3. Define convergence series with example.
4. Find the coefficient of  $x^6$  in the expansion of  $(1 - x + x^2)e^{2x}$ .
5. Find the radius of curvature of the curve  $xy = c^2$  at  $(c, c)$ .
6. Find the envelope of the family of straight lines  $y = mx + \frac{a}{m}$ ,  $m$  being the parameter.
7. Find  $\frac{du}{dt}$  when  $u = x^2 + y^2$ ,  $x = at^2$ ,  $y = 2at$ .
8. If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , find  $\frac{\partial(r, \theta)}{\partial(x, y)}$ .
9. Evaluate  $\int_0^1 \int_0^2 \int_0^3 xyz \, dx \, dy \, dz$ .
10. Change the order of integration in  $\int_0^1 \int_0^y f(x, y) \, dx \, dy$ .

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix  $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$ . (8)

- (ii) Using Cayley-Hamilton theorem find  $A^{-1}$ , where  $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$ . (8)

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- (b) Reduce the quadratic form  $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$  to canonical form. (16)

12. (a) (i) Examine the convergence of the series  $\frac{1}{2!} - \frac{2}{3!} + \frac{3}{4!} \dots \infty$ . (8)

- (ii) Find the sum to infinity of the series  $\frac{1}{1!} + \frac{1+5}{2!} + \frac{1+5+5^2}{3!} + \dots \infty$ . (8)

Or

- (b) (i) Expand  $\frac{1}{(1-2x)^2(1-3x)}$  in ascending powers of  $x$ . Also find the coefficient of  $x^n$ . (8)

- (ii) Prove that  $\sqrt{x^2+4} - \sqrt{x^2+1} = 1 - \frac{x^2}{4} + \frac{7}{64}x^4$  nearly when  $x$  is small. (8)

13. (a) (i) Find the equation of the circle of curvature of the parabola  $y^2 = 12x$  at (3, 6). (8)

- (ii) Find the equation of evolute of the curve  $x = a(\cos t + t \sin t)$ ,  $y = a(\sin t - t \cos t)$ . (8)

Or

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- (b) (i) Find the radius of curvature at  $(a, 0)$  on the curve  $xy^2 = a^3 - x^3$ . (8)

- (ii) Find the envelope of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , where  $a$  and  $b$  are connected by the relation  $a^2 + b^2 = c^2$ ,  $c$  being a constant. (8)

14. (a) (i) If  $u = f(r, s, t)$  and  $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$ , find the value of

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}. \quad (8)$$

(ii) Examine the extrema of  $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$ . (8)

Or

(b) (i) Using Taylor's series expansion, expand  $e^x \sin y$  in powers of  $x$  and  $y$  as far as terms of the 3<sup>rd</sup> degree. (8)

(ii) Find the shortest and longest distances from the point  $(1, 2, -1)$  to the sphere  $x^2 + y^2 + z^2 = 24$ . (8)

15. (a) (i) Evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2} \, dx \, dy$ . (8)

(ii) Using double integral find the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . (8)

Or

(b) (i) Change the order of integration in  $\int_0^{2\sqrt{4-y^2}} \int_0^y xy \, dx \, dy$  and evaluate it. (8)

(ii) By transforming into polar co-ordinates evaluate  $\iint_0^{2\pi} \int_0^{\infty} e^{-(x^2+y^2)} \, dx \, dy$ .

Hence find the value of  $\int_0^{\infty} e^{-x^2} \, dx$ . (8)