Reg. No.

Question Paper Code : 57495

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

First Semester

Mechanical Engineering

MA 6151 – MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A ($10 \times 2 = 20$ Marks)

1. If the eigen values of the matrix A of order 3×3 are 2, 3 and 1, then find the eigen values of adjoint of A.

2. If λ is the eigen value of the matrix A, then prove that λ^2 is the eigen value of A^2.

3. Give an example for conditionally convergent series.

4. Test the convergence of the series $1 - \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{7^2} - \ldots\ldots\ldots$

5. Define evolutes of the curve.

6. Find the envelope of the family of curves $y = mx + \frac{1}{m}$, where m is the parameter.

7. If $x^2 + y^2 = 1$, then find $\frac{dy}{dx}$.

8. If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(r, \theta)}{\partial(x, y)}$

9. Sketch the region of integration in $\int_0^1\int_0^x dy\, dx$.

10. Find the area bounded by the lines $x = 0$, $y = 1$, $x = 1$ and $y = 0$.

PART – B ($5 \times 16 = 80$ Marks)

11. (a) (i) Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (8)

(ii) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. Hence using it find A^{-1}. (8)

OR

(b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Hence find its rank and nature. (16)

12. (a) (i) Discuss the convergence and the divergence of the following series :

$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\text{to } \infty.$ (8)

(ii) Find the interval of the convergence of the series : $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \ldots$ (8)

OR

(b) (i) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{n}\right)$. **(8)**

(ii) Test the convergence of the series $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \ldots\ldots$ to ∞ **(8)**

13. (a) (i) Find the equation of circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$. **(8)**

(ii) Find the equation of the evolutes of the parabola $y^2 = 4ax$. **(8)**

OR

(b) (i) Find the radius of curvature at t on $x = e^t \cos t$, $y = e^t \sin t$. **(8)**

(ii) Find the envelope of the family of straight lines $y = mx - 2\,am - am^3$, where m is the parameter. **(8)**

14. (a) (i) Expand $e^x \log(1 + y)$ in powers of x and y up to the third degree terms using Taylor's theorem. **(8)**

(ii) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r\cos\theta$, find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. **(8)**

OR

(b) (i) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction. **(8)**

(ii) If $w = f(y - z, z - x, x - y)$, then show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. **(8)**

15. (a) (i) By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$. **(8)**

(ii) By changing to polar co-ordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$. **(8)**

OR

(b) (i) Evaluate $\iint xy \, dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. **(8)**

(ii) Evaluate $\iiint_V \frac{dz dy dx}{(x+y+z+1)^3}$, where V is the region bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. **(8)**

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