Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 77184

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

First Semester

Mechanical Engineering

MA 6151 — MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the sum and product of all the eigen values of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

2. Give the nature of a quadratic form whose matrix is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

3. Distinguish between a sequence and series.

4. State the Integral test.

5. What is circle of curvature?

6. Find the envelope of $x.\cos\theta + y.\sin\theta = 1$, where θ is a parameter.

7. If $u = x^2 + y^2$ and $x = at^2$, $y = 2at$, find $\frac{du}{dt}$.

8. State the conditions for maxima and minima of $f(x, y)$.

9. Evaluate : $\int_1^2 \int_1^3 \frac{dxdy}{xy}$.

10. Obtain the value of $\int_0^a \int_0^b \int_0^c dxdydz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$. (8)

(ii) If $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$, verify Cayley- Hamilton theorem and hence find A^{-1}. (8)

Or

(b) Reduce the quadratic form $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$ into canonical form and hence find its rank. (16)

12. (a) (i) Using comparison test, examine the convergence or divergence of $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + ...$ (8)

(ii) Using D'Alembert's ratio test, examine the convergence or divergence of $x + 2x^2 + 3x^3 + ...$ (8)

Or

(b) (i) Test for convergence or divergence of $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + ...$ (8)

(ii) Test for absolute convergence of $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ (8)

13. (a) (i) Find the radius of curvature of $x^{2/3} + y^{2/3} = a^{2/3}$. (8)

(ii) Obtain the evolute of $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$. (8)

Or

(b) (i) Find the centre of curvature of $x^3 + y^3 = 6xy$ at (3,3). (8)

(ii) Obtain the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, if $a^2 + b^2 = c^2$. (8)

14. (a) (i) If $u = \log(\tan x + \tan y + \tan z)$, find $\sum \sin 2x.\frac{\partial u}{\partial x}$. (8)

(ii) Obtain the Taylor series of $x^3 + y^3 + xy^2$ in powers of $x - 1$ and $y - 2$. (8)

Or

(b) (i) Find the Jacobian of $u = x + y + z$, $v = xy + yz + zx$, $w = x^2 + y^2 + z^2$. (8)

(ii) Obtain the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (8)

15. (a) (i) By changing the order of integration, evaluate : $\int_0^1 \int_y^1 \frac{x}{x^2 + y^2} dxdy$. (8)

(ii) Find the volume of $x^2 + y^2 + z^2 = r^2$ using triple integral. (8)

Or

(b) (i) Using double integral, find the area of $r = a(1 + \cos\theta)$. (8)

(ii) Evaluate : $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$. (8)
