USN | | | | | | | | | | | 10MAT31

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016

Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

PART – A

1 a. For the function :

$$f(x) = \begin{cases} x & \text{in} \quad 0 < x < \pi \\ x - 2\pi & \quad \pi < x < 2\pi \end{cases}$$

Find the Fourier series expansion and hence deduce the result $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \cdots\cdots$. **(07 Marks)**

b. Obtain the half range Fourier cosine series of the function $f(x) = x(\ell - x)$ in $0 \le x \le \ell$. **(06 Marks)**

c. Find the constant term and first harmonic term in the Fourier expansion of y from the following table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

2 a. Find the Fourier transform of the function :

$f(x) = \begin{cases} 1 & \text{for} \quad |x| \le a \\ 0 & \text{for} \quad |x| > a \end{cases}$ and hence evaluate : $\int_0^\infty \frac{\sin x}{x} dx$. **(07 Marks)**

b. Obtain the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1 + x^2} dx, m > 0$. **(06 Marks)**

c. Solve the integral equation : $\int_0^\infty f(x) \cos px\, dx = \begin{cases} 1 - p, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases}$ and hence deduce the value of $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. **(07 Marks)**

3 a. Obtain the various possible solutions of the two dimensional Laplace's equation $u_{xx} + u_{yy} = 0$ by the method of separation of variables. **(07 Marks)**

b. A string is stretched and fastened to two points 'ℓ' apart. Motion is started by displacing the string in the form $y = a \sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at time t = 0. Show that the displacement of any point at a distance 'x' from one end at time 't' is given by $y(x, t) = a \sin\left(\frac{\pi x}{\ell}\right) \cos\left(\frac{\pi ct}{\ell}\right)$. **(06 Marks)**

c. Obtain the D' Alembert's solution of the wave equation $u_{tt} = c^2 u_{xx}$ subject to the conditions $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = a$. **(07 Marks)**

4 a. For the following data fit an exponential curve of the form $y = a e^{bx}$ by the method of least squares :

x	5	6	7	8	9	10
y	133	55	23	7	2	2

(07 Marks)

b. Solve the following LPP graphically :
Minimize $Z = 20x + 10y$
Subject to the constraints : $x + 2y \le 40$
$3x + y \ge 30$
$4x + 3y \ge 60$
$x \ge 0$ and $y \ge 0$. (06 Marks)

c. Using Simplex method, solve the following LPP :
Maximize : $Z = 2x + 4y$
Subject to the constraints $3x + y \le 22$
$2x + 3y \le 24$
$x \ge 0$ and $y \ge 0$. (07 Marks)

PART – B

5 a. Using the Regula – Falsi method to find the fourth root of 12 correct to three decimal places. (07 Marks)

b. Apply Gauss – Seidal method, to solve the following of equations correct to three decimal places :
$6x + 15y + 2z = 72$
$x + y + 54z = 110$
$27x + 6y - z = 8.5$
(carry out 3 iterations). (06 Marks)

c. Using Rayleigh power method, determine the largest eigen value and the corresponding eigen vector, of the matrix A in six iterations. Choose $[1\ 1\ 1]^T$ as the initial eigen vector :
$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$
(07 Marks)

6 a. Using suitable interpolation formulae, find y(38) and y(85) for the following data :

x	40	50	60	70	80	90
y	184	204	226	250	276	304

(07 Marks)

b. If $y(0) = -12$, $y(1) = 0$, $y(3) = 6$ and $y(4) = 12$, find the Lagrange's interpolation polynomial and estimate y at $x = 2$. (06 Marks)

c. By applying Weddle's rule, evaluate : $\int_0^1 \frac{x\,dx}{1+x^2}$ by considering seven ordinates. Hence find the value of $\log_e^2$. (07 Marks)

7 a. Using finite difference equation, solve $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to $u(0, t) = u(4, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ upto four time steps. Choose $h = 1$ and $k = 0.5$. **(07 Marks)**

b. Solve the equation $u_t = u_{xx}$ subject to the conditions $u(0, t) = 0$, $u(1, t) = 0$, $u(x, 0) = \sin(\pi x)$ for $0 \le t \le 0.1$ by taking $h = 0.2$. **(06 Marks)**

c. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown. Find the first iterative values of $u_i (i = 1 - 9)$ to the nearest integer. **(07 Marks)**

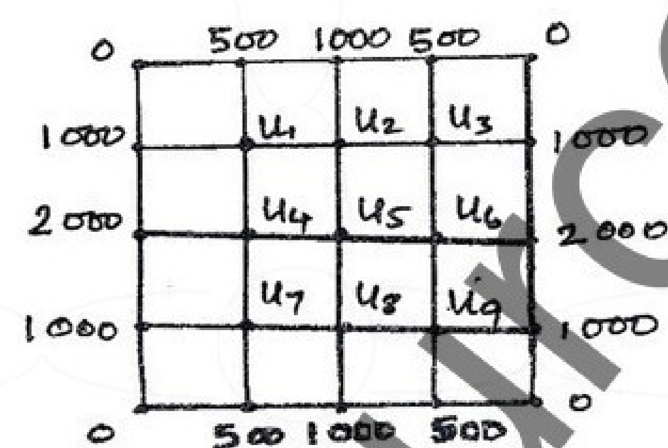

Fig.Q7(c)

8 a. Find the z – transform of $2n + \sin(n\pi/4) + 1$. **(07 Marks)**

b. Obtain the inverse z – transform of $\frac{2z^2 + 3z}{(z + 2)(z - 4)}$. **(06 Marks)**

c. Using z – transform, solve the following difference equation :
$u_{n+2} + 2u_{n+1} + u_n = n$ with $u_0 = u_1 = 0$. **(07 Marks)**

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