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10CS42

Fourth Semester B.E. Degree Examination, June/July 2017

Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: *Answer FIVE full questions, selecting at least TWO questions from each part.*

PART – A

1 a. Given a graph G, shown in Fig.Q1(a) determine:
i) Number of paths from a to h
ii) Number of paths from a to h with path length 5.

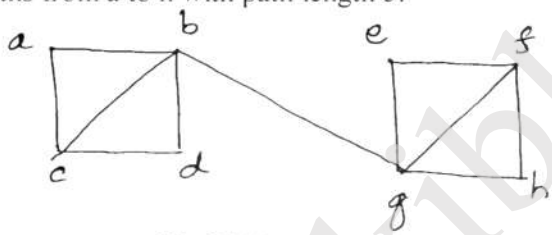

Fig.Q1(a) **(06 Marks)**

b. Prove that the maximum number of edges in a simple graph with n vertices is $n(n-1)/2$. **(06 Marks)**

c. Define isomorphism of graphs. Determine whether the following graphs are isomorphic.

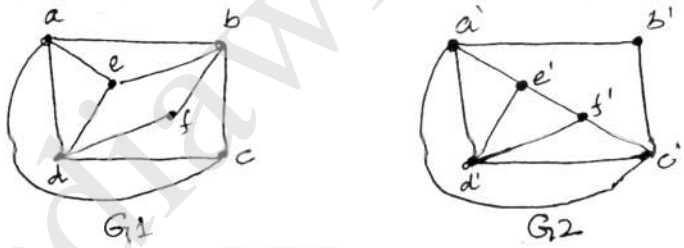

Fig.Q1(c) **(06 Marks)**

d. Draw a 3-regular graph which has 10 vertices and 15 edges. **(02 Marks)**

2 a. State and prove the Euler's theorem for a connected planar graph G with n vertices , m edges and number of regions r. **(06 Marks)**

b. Show that in a complete graph with n vertices where n is an odd number and $n \geq 3$ there are $(n-1)/2$ edge Hamilton cycles. **(06 Marks)**

c. Define chromatic number of a graph. Determine the chromatic polynomial for the graph G shown in Fig.Q2(c).

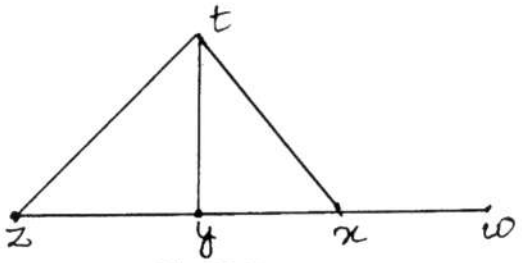

Fig.Q2(c) **(08 Marks)**

3 a. Answer the following questions for the tree shown in Fig.Q3(a).
   i) Which vertices are the descendants of C?
   ii) Which vertices are the siblings of S?
   iii) Which vertices have level number 4?

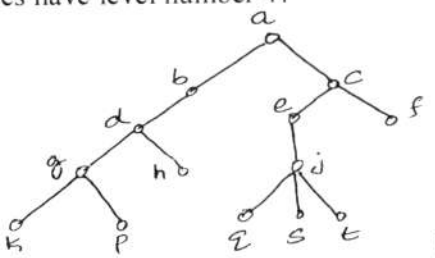

Fig.Q3(a) **(03 Marks)**

b. Define the following with respect to a rooted tree with root r T = (V, E).
   i) Preorder traversal of T
   ii) Post-order traversal of T.
   Given rooted tree shown in Fig.Q3(b). Find preorder traversal and post order traversal.

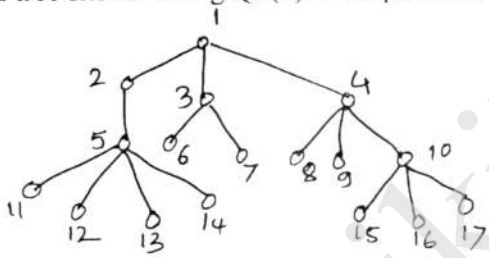

Fig.Q3(b) **(08 Marks)**

c. i) Prove that in every tree T = (V, E), |V| = |E| + 1.
   ii) Let $F_1 = (V_1, E_1)$ be a forest of 7 trees, where $|E_1| = 40$. What is $|V_1|$? **(09 Marks)**

4 a. Define by giving an example for each:
   i) Matching
   ii) Complete matching
   iii) Edge connectivity **(06 Marks)**

b. Explain the max-flow min-cut theorem. For the given weighted graph shown in the Fig.Q4(b) below, find all possible cut-sets from the vertices A and E, and hence find minimum and maximum capacity.

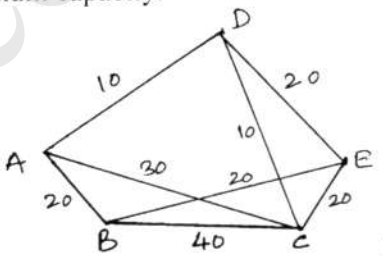

Fig.Q4(b) **(06 Marks)**

c. Give Kruskal's algorithm for an optimal spanning tree. Hence find a minimal spanning tree for the graph shown in Fig.Q4(c).

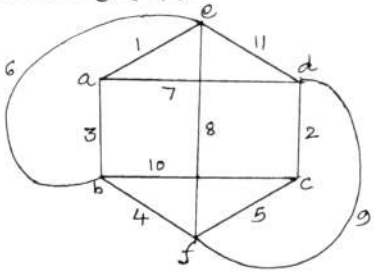

Fig.Q4(c) **(08 Marks)**

PART – B

5 a. A minimeals include a soup, a course, a dessert and an ice-cream. Suppose that a customer can select from 5 soups, 6 course, 4 desserts and 3 types of ice-creams. How many different minimeals can be selected? **(06 Marks)**

b. A certain question paper contains two parts A and B each having 4 questions. How many different ways a student can answer 5 questions by selecting atleast two questions from each part? **(06 Marks)**

c. In how many possible ways could a student answer a 10 question TRUE/FALSE test? **(02 Marks)**

d. Show that $b_{n+1} = \frac{2(2n+1)}{(n+2)} \times b_n$ where b_n is the n^{th} Catalan number. **(06 Marks)**

6 a. In how many ways can integers 1, 2, 3, ……,10 be arranged in a line so that no even integer is in its natural place? **(10 Marks)**

b. A girl student has sarees of 5 different colors: blue, green, red, white and yellow. On Monday she does not wear green, on Tuesday blue or red, on Wednesday blue or green, on Thursday red or yellow and on Friday red. In how many ways can she dress without repeating a color during a week from Monday to Friday? **(10 Marks)**

7 a. In how many ways can 12 oranges can be distributed among 3 children A, B, and C so that A gets at least 4, B and C get at least 2 but C gets not more than five? **(10 Marks)**

b. Determine the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$. **(10 Marks)**

8 a. Find and solve a recurrence relation for the number of binary sequences of length $n \geq 1$ that have no consecutive 0's. **(10 Marks)**

b. The number of virus affected files in a system is 1000 (to start with) and this increases 250% in every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(10 Marks)**

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