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10CS34

Third Semester B.E. Degree Examination, Dec.2014/Jan.2015
Discrete Mathematical Structures

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting atleast TWO questions from each part.

PART – A

- 1 a. Simply the set expression $\overline{((A \cup B) \cap C \cup \overline{B})}$ with justification. (06 Marks)
- b. i) Use membership table to establish the set equality of:
 $\overline{(A \cap B) \cup (\overline{A} \cap C)} = (A \cap \overline{B}) \cup (\overline{A} \cap \overline{C})$. (07 Marks)
- ii) If $A = \{1, 2, 3, 4, 5, 6, 7\}$, determine the number of subsets of A containing 3 elements, subsets of A containing 1, 2, and subsets of A with even number of elements.
- c. The sample space of an experiment is $S = \{a, b, c, d, e, f, g, h\}$. If event $A = \{a, b, c\}$ and event $B = \{a, c, e, g\}$, determine $P_r(A)$, $P_r(B)$, $P_r(A \cap B)$, $P_r(A \cup B)$, $P_r(\overline{A})$, $P_r(A \cap \overline{B})$ and $P_r(\overline{A \cup B})$. (07 Marks)

- 2 a. Construct truth table for :
 i) $[p \wedge (p \rightarrow q)] \rightarrow q$
 ii) $[p \rightarrow q] \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$. (06 Marks)
- b. Simplify the switching network using the laws of logic. (07 Marks)

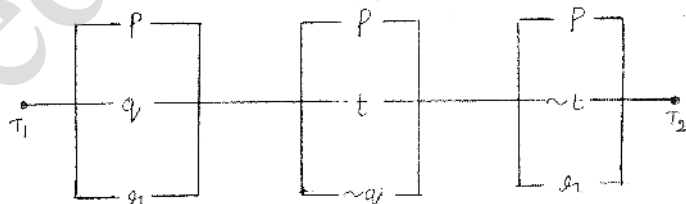


Fig. Q2(b)

- c. Establish the following argument by the methods of proof by contradiction.
 $P \rightarrow (q \wedge r)$
 $r \rightarrow s$
 $\sim (q \wedge s)$
 $\therefore \sim p$ (07 Marks)

- 3 a. Negate and simplify each of the following :
 i) $\exists x, [p(x) \vee q(x)]$
 ii) $\forall x, [p(x) \wedge \sim q(x)]$
 iii) $\forall x, [p(x) \rightarrow q(x)]$
 iv) $\exists x, [(p(x) \vee q(x)) \rightarrow r(x)]$. (06 Marks)
- b. Find whether the following argument is valid. No engineering student of first and second semester studies logic.
Anil is an engineering student who studies logic
 \therefore Anil is not in second semester (07 Marks)
- c. Give : i) a direct proof ii) an indirect proof and iii) proof by contradiction for the following statement. "If m is an even integer, then m + 7 is odd". (07 Marks)

- 4 a. If $H_1 = 1, H_2 = 1 + \frac{1}{2} + \dots, H_n = 1 + \frac{1}{2} + \dots + \frac{1}{n}$ are harmonic numbers, then prove that for all $n \in \mathbb{Z}^+$

$$\sum_{i=1}^n H_i = (n+1)H_n - n.$$

(06 Marks)

- b. For all $n \in \mathbb{Z}^+$ prove that :

$$\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$$

$$\sum_{i=1}^n i(2^i) = 2 + (n-1)2^{n+1}.$$

(07 Marks)

- c. i) If $A_1, A_2, \dots, A_n \subseteq U$, then prove that :

$$A_1 \cap A_2 \cap \dots \cap A_n = \overline{A_1 \cup A_2 \cup \dots \cup A_n}$$

- ii) If $A, B_1, B_2, \dots, B_n \subseteq U$ then prove that $A \cap (B_1 \cup B_2 \cup \dots \cup B_n) = (A \cap B_1) \cup (A \cap B_2) \cup \dots \cup (A \cap B_n)$.

(07 Marks)

PART - B

- 5 a. Find the number of ways of distributing 6 objects among 4 identical containers with some containers possibly empty. (06 Marks)

- b. (i) Prove that the function $f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(a, b) = \lceil a + b \rceil$ is commutative but not associative

- (ii) Prove that if 30 dictionaries in a library contains a total of 61,327 pages, then atleast one of the dictionary must have atleast 2045 pages. (07 Marks)

- c. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$$

- then determine $f^{-1}(-1), f^{-1}(3), f^{-1}(6), f^{-1}(-5, 5)$. (07 Marks)

- 6 a. Give a set A with $|A| = n$ and a relation R on A , let M denote the relation matrix for R then prove that :

- i) R is symmetric if and only if $M = M^t$

- ii) R is transitive if and only if $M.M = M^2 \leq M$. (06 Marks)

- b. Let $A = \{1, 2, 3, 4, 6, 8, 12\}$ and R be the partial ordering on A defined by $a R_b$ if a divides b then

- i) Draw the Hasse diagram of the Poset (A, R)

- ii) Determine the relation matrix for R

- iii) Topologically sort the Poset (A, R) (07 Marks)

- c. Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$, and define R on A by $(x_1, y_1) R (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$

- i) Verify that R is an equivalence relation on A

- ii) Determine the equivalence classes $[(1, 3)], [(2, 4)]$ and $[(1, 1)]$

- iii) Determine the partition of A induced by R . (07 Marks)

7 a. In a group S_6 , let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 6 & 2 & 5 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 2 & 6 & 1 & 3 & 5 \end{pmatrix}$$

(06 Marks)

Determine $\alpha\beta$, α^3 , β^4 , $(\alpha\beta)^{-1}$.

b. Define cyclic group and prove that every subgroup of a cyclic group is cyclic. (07 Marks)

c. Define the coding function $E: Z_2^3 \rightarrow Z_2^6$ by means of parity - check matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \text{ and determine all code words. (07 Marks)}$$

8 a. Let $E: Z_2^m \rightarrow Z_2^n$ be an encoding function given by a generator matrix G or the associated parity check matrix H then prove that $C = E(Z_2^m)$ is a group code. (06 Marks)

b. i) Define subring and ideal.

ii) If $A = \left\{ \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \mid a, b, c \in Z \right\}$ be the subset of the ring $R = M_2(Z)$ then prove that A is a subring but not ideal. (07 Marks)

c. i) Prove that Z_n is a field if and only if n is a prime.

ii) Prove that in Z_n , $[a]$ is a unit if and only if $\gcd(a, n) = 1$. (07 Marks)
