Third Semester B.E. Degree Examination, Dec.2014/Jan.2015
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Expand $f(x)=\sqrt{1-\cos x}$, $0 < x < 2\pi$ in a Fourier series. Hence evaluate $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots$ **(07 Marks)**

b. Find the half-range sine series of $f(x) = e^x$ in (0, 1). **(06 Marks)**

c. In a machine the displacement y of a given point is given for a certain angle x as follows:

x	0	30	60	90	120	150	180	210	240	270	300	330
y	7.9	8	7.2	5.6	3.6	1.7	0.5	0.2	0.9	2.5	4.7	6.8

Find the constant term and the first two harmonics in Fourier series expansion of y. **(07 Marks)**

2 a. Find Fourier transform of $e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{\cos xt}{1+t^2}dt$. **(07 Marks)**

b. Find Fourier sine transform of $f(x)=\begin{cases} x, & 0<x\le 1 \\ 2-x, & 1\le x<2 \\ 0, & x>2 \end{cases}$. **(06 Marks)**

c. Solve the integral equation $\int_0^\infty f(x)\cos\lambda x\,dx = e^{-\lambda}$. **(07 Marks)**

3 a. Find various possible solution of one-dimensional heat equation by separable variable method. **(10 Marks)**

b. A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge y = 0 is given by

$u = 20x,\ 0 \le x \le 5$

$\ \ = 20(10-x),\ 5 \le x \le 10$

and the two long edges x = 0, x = 10 as well as the other short edge are kept at 0°C. Find the temperature u(x, y). **(10 Marks)**

4 a. Fit a curve of the form $y = ae^{bx}$ to the data: **(07 Marks)**

x	1	5	7	9	12
y	10	15	12	15	21

b. Use graphical method to solve the following LPP:

Minimize $Z = 20x_1 + 30x_2$

Subject to $x_1 + 3x_2 \ge 5$;

$2x_1 + 2x_2 \ge 20$;

$3x_1 + 2x_2 \ge 24$;

$x_1, x_2 \ge 0$. **(06 Marks)**

c. Solve the following LPP by using simplex method:

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1 \ge 0, x_2 \ge 0.$ **(07 Marks)**

PART – B

5 a. Use the Gauss-Seidal iterative method to solve the system of linear equations. $27x + 6y - z = 85$; $6x + 15y + 2z = 72$; $x + y + 54z = 110$. Carry out 3 iterations by taking the initial approximation to the solution as (2, 3, 2). Consider four decimal places at each stage for each variable. **(07 Marks)**

b. Using the Newton-Raphson method, find the real root of the equation $x\sin x + \cos x = 0$ near to $x = \pi$, carryout four iterations (x in radians). **(06 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the matrix

$A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{pmatrix}$ by power method. Take $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ as the initial vector. Perform 5 iterations. **(07 Marks)**

6 a. Find f(0.1) by using Newton's forward interpolation formula and f(4.99) by using Newton's backward interpolation formula from the data: **(07 Marks)**

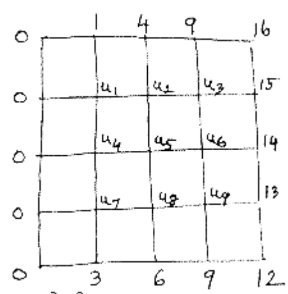
x	0	1	2	3	4	5
f(x)	-8	0	20	58	120	212

b. Find the interpolating polynomial f(x) by using Newton's divided difference interpolation formula from the data: **(06 Marks)**

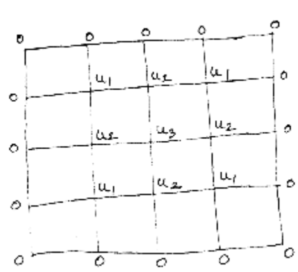
x	0	1	2	3	4	5
f(x)	3	2	7	24	59	118

c. Evaluate $\int_0^{1.2} e^x\, dx$ using Weddle's rule. Taking six equal sub intervals, compare the result with exact value. **(07 Marks)**

7 a. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the following square mesh. Carry out two iterations. **(07 Marks)**



b. Solve the Poisson's equation $\nabla^2 u = 8x^2y^2$ for the square mesh given below with u = 0 on the boundary and mesh length, h = 1. **(06 Marks)**



c. Evaluate the pivotal values of $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$ taking h = 1 upto t = 1.25. The boundary conditions are $u(0, t) = 0$, $u(5, t) = 0$, $\frac{\partial u}{\partial t}(x, 0) = 0$, $u(x, 0) = x^2(5 - x)$. **(07 Marks)**

8 a. Find the Z-transforms of i) $\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n$ ii) $3^n \cos\frac{\pi n}{4}$. **(07 Marks)**

b. State and prove initial value theorem in Z-transforms. **(06 Marks)**

c. Solve the difference equation
$u_{n+2} - 2u_{n+1} + u_n = 2^n$; $u_0 = 2, u_1 = 1$. **(07 Marks)**

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